PEARL Inc.

Web: http://www.pearl-hifi.com 86008, 2106 33 Ave. SW, Calgary, AB; CAN T2T 1Z6
E-mail: custserv@pearl-hifi.com Ph: +.1.403.244.4434 Fx: +.1.403.245.4456

Perkins Electro-Acoustic Research Lab, Inc. ❦ Engineering and Intuition Serving the Soul of Music

Distortion in Transformer Cores

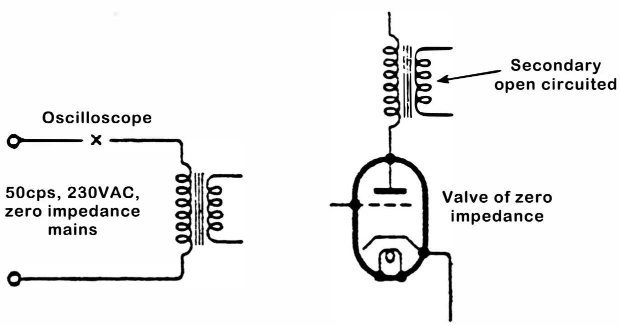
Part I. OSCILLOGRAMS AND WHAT THEY REVEAL

By N. PARTRIDGE. B.Sc. (Eng.) A.M.I.E.E.

THERE are two distinct types of distortion in audio-frequency apparatus, (1) frequency distortion and (2) harmonic or amplitude distortion. To achieve ultra-high-quality reproduction it is necessary to study and eliminate both. Obviously, it would be useless to construct an amplifier having a perfect frequency characteristic if it produced, say, 10 per cent. harmonic distortion at normal volume! Yet, strange as it is, this very one-sided treatment seems to be accepted as adequate when referring to output transformers.

That iron introduces harmonic distortion has been known for a very long time. Textbooks and periodicals, not omitting *The Wireless World*, make frequent, if obscure, references to it. "The core must be operated at a low flux density"; "a large core should be employed," etc., are typical observations. But how low must be the flux density and how big the core? Above all, what percentage harmonic distortion normally occurs in an output transformer? A little reflection will bring home the fact that such figures are never called into question. Harmonic distortion has been completely overshadowed by considerations of frequency response.

THIS is the first of a series of articles in which the question of amplitude distortion arising in the iron core of a transformer will be dealt with on a quantitative basis. Information on this subject is scarce and the design data given is the outcome of original research by the author.



(a) (b)

Fig. 1. At (a) is shown an easy method of observing distortion of the current flowing in the primary of a transformer, while (b) illustrates an equivalent valve circuit.

The simplest way of making the acquaintance of iron distortion is to connect the primary of an output transformer across the 230 v. 50 c/s mains with an oscilloscope in series with it as in Fig. 1 (a). A typical oscillogram is reproduced in Fig. 2.

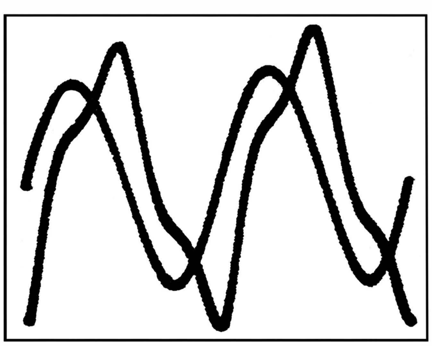


Fig. 2. A typical oscillogram obtained from the circuit arrangement of Fig. 1(a).

The sine wave represents the mains voltage applied to the transformer, and the distorted curve represents the current flowing through it. The latter lags behind the voltage, as would be expected with an inductive load, and is badly out of shape. True, the circuit is not the equivalent of that in which output transformers are usually employed. First, the secondary is not loaded, and, secondly, the mains must be regarded as having zero impedance. Fig. 1(b) indicates the corresponding valve circuit. But although the arrangement is not representative of the usual conditions of operation, it will be seen later that it provides a key to all cases. The next step, therefore, is to put our somewhat haphazard experiment upon a more scientific footing.

There is a well-known formula which is as follows:

$$B = \frac{\text{Volts} \times 10^3}{4.44 \times \text{Frequency} \times \text{Core area} \times \text{Turns}} \qquad (1)$$

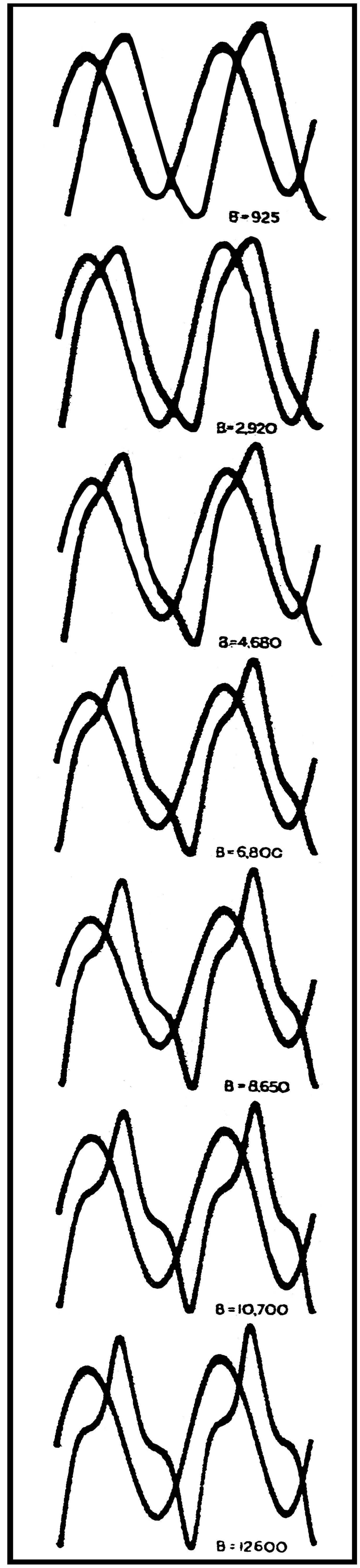


Fig. 3. A series of oscillograms illustrating how current distortion varies with flux density; 'B' is the value of the peak flux density. The core material was Silcor 2.

"B" is the peak value of the flux density reached in the core during a complete cycle. All the constants on the right-hand side of the equation are easily determined. The volts will be 230 or whatever the mains voltage happened to be when the photograph in Fig. 2 was taken, the frequency 50 c/s, the core area (in sq. cms.) can be measured and the turns upon the primary of the transformer can be counted. Thus the peak flux density is readily deduced and our photo is at once elevated from a typical example of distortion to a specific instance of distortion at a known flux density.

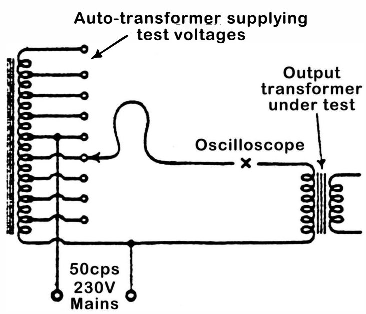


Fig. 4. A modification of the circuit of Fig. 1; (a) that allows observations of distortion at various known flux densities.

An extension of this idea can be added by turning formula (1) round the other way, thus:

$$\text{Volts} = \frac{4.44 \times B \times \text{Freq.} \times \text{Core area} \times \text{Turns}}{10^3} \qquad (2)$$

By selecting a number of values of "B" for which we should like distortion curves, the required test voltages can be calculated. An auto-transformer will serve to step the mains voltage up or down as necessary and enable us to produce a range of current oscillograms showing how the distortion varies with different flux densities. Fig. 4 outlines the circuit suggested. It is very important to note that a transformer *must* be used for regulating the test voltage. If a series resistance or potentiometer were employed it would act as a series impedance and render the results inconsistent.

Working on these lines, the oscillograms reproduced in Fig. 3 were obtained. The sine curves, in phase with the voltage, were taken for calibration purposes, and can be ignored by the reader; the distorted current curves are all that matter for the moment. The maximum flux density is marked on each photograph. The particular material used for the core of the transformer was Silcor 2, supplied by Messrs. Magnetic and Electrical Alloys, Ltd., of Wembley. This grade of iron is commonly employed for speech transformers, and can be taken as representative of general practice. Obviously, different grades of iron are likely to produce different degrees of distortion. This aspect of the problem will receive full attention later.

With a modicum of mathematical knowledge and unlimited patience one can analyse the curves of Fig. 3 and produce a graph showing the magnitude of the various harmonics at all flux densities. Fig. 5 gives the result of this labour, and, as will be seen in due course, is a highly important addition to our knowledge of the characteristics of magnetic materials.

The harmonics are expressed as a percentage of the fundamental i.e., of the true 50 c/s current. Only odd harmonics are present, as would be expected from the symmetrical nature of the oscillograms. Even harmonics result in an unsymmetrical wave shape. The analysis was in all cases extended to the eleventh harmonic, but only the third, fifth, and seventh were found to be of importance.

The meaning of Fig. 5 must be thoroughly understood before proceeding. It shows the harmonics, expressed as a percentage of the fundamental, present in the current flowing through an unloaded transformer when the said transformer is connected across a low-impedance source of such voltage and periodicity as to produce the corresponding flux density. The figures give no direct indication of the distortion the transformer would produce in normal use, but rather represent a characteristic of the core material. We have to find out how these figures may be used as a basis for calculating the performance of the transformer in any specified circuit.

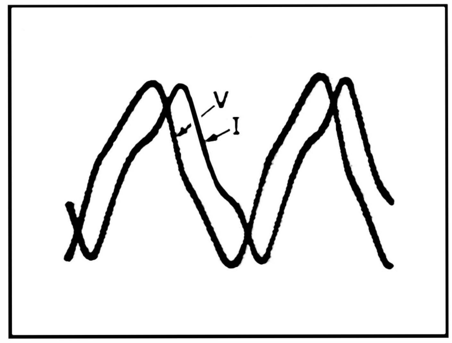


Fig. 6. Showing how a medium impedance, such a valve output stage, in series with the transformer causes both current and voltage to be distorted.

Increasing the impedance of the AC source i.e., the mains supply—would bring conditions more nearly into line with those found in practice. Normally, a valve can have an AC impedance of anything from a fraction up to many times that of the transformer primary. The former case occurs when a low-impedance triode is used, and the latter when high-impedance tetrodes or pentodes are employed.

Fig. 6 shows the effect of introducing an impedance of approximately equal value in series with the transformer primary. The flux density was, in this instance, approximately 7,000 lines per sq. cm., and the photograph may be compared with the corresponding oscillogram in Fig. 3. Note that both voltage and current have become distorted. This is to be expected, because the transformer draws a distorted current, and therefore the voltage drop across the series impedance must of necessity be distorted. Hence the voltage across the transformer, which is the mains voltage minus the distorted drop across

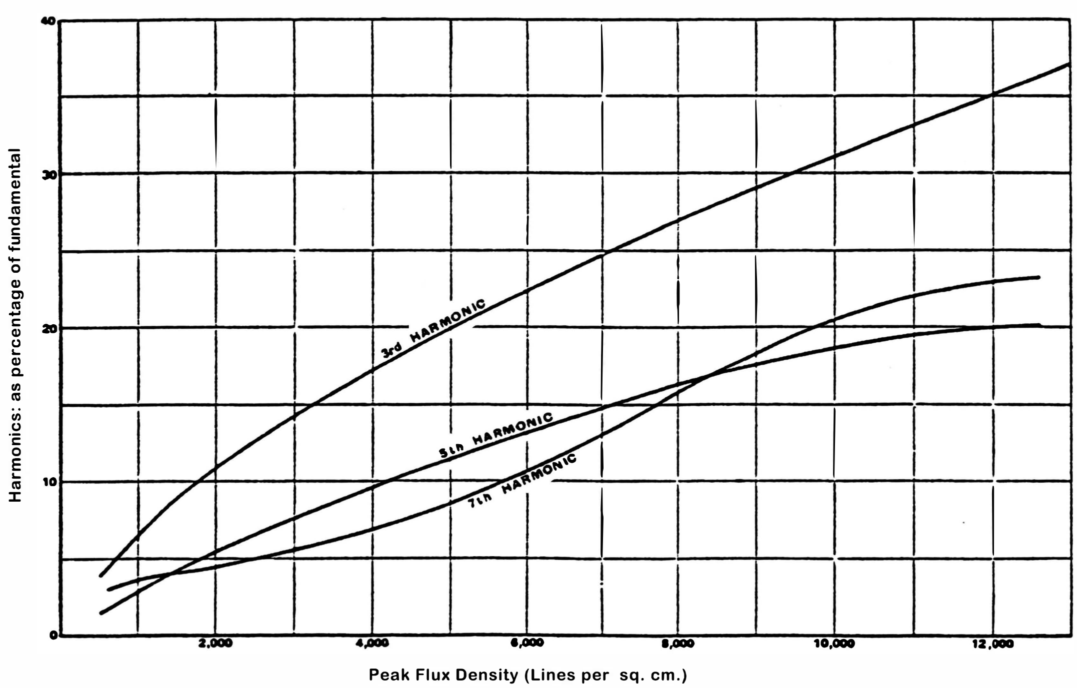


Fig. 5. The graph obtained by analyzing the oscillograms of Fig. 4. The harmonics are expressed as percentages of the fundamental.
This is a very important series of curves and forms the basis of transformer distortion calculations.

the series impedance, must also be distorted.

Evidently an extreme case could be arrived at by making the series impedance very high compared with that of the transformer. Fig. 7 shows what happens under these circumstances. The current becomes a pure sine wave, and the distortion is transferred to the voltage curve. The flux density during this experiment was again maintained at a value of about 7,000 lines per sq. cm. (See Appendix.)

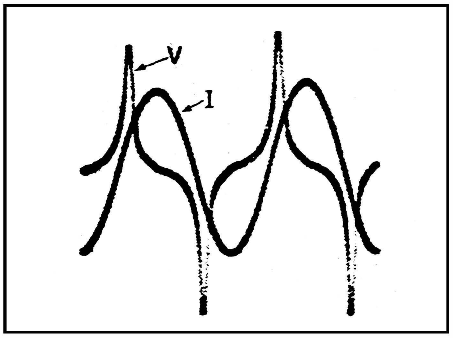


Fig. 7. A very high impedance, such as a pentode output stage, results in distortion of the voltage while yielding a sine wave current waveform.

To sum up, at one extreme, where the impedance of the AC source is zero, the current is distorted while the voltage remains a pure sine curve. At the other extreme, where the source has a high relative impedance, the reverse is true i.e., the voltage is distorted and the current a pure sine wave. Clearly, the relative impedance of the transformer compared with that of its associated valve is of considerable importance. A valve impedance is reasonably constant and can be estimated from figures supplied by the manufacturers. But the impedance of a transformer, even at a fixed frequency, is anything but constant, and we must look into this question more closely.

A good many years ago it was the custom of certain manufacturers, and, I believe, of *The Wireless World* as well, to state the inductance of chokes and transformers when *tested at a specified alternating terminal voltage*. This was a very excellent idea, but it has died out, and only the inductance figure is mentioned nowadays. One tends to imagine the inductance of an output transformer as a fixed and constant thing, rather like the resistance of a piece of wire or the capacity of a condenser. Actually, it is entirely dependent upon the test voltage, or, what comes to the same thing, the flux density. A test taken at around 5,000 lines per sq. cm. will give an inductance value perhaps three times as great as that measured at 100 lines per sq. cm.

Impedance Calculations

Definite figures can readily be obtained by replacing the oscillograph shown in the circuit of Fig. 4 by an ammeter. We have already seen how to work out the flux densities corresponding to the several tappings on the auto transformer by using formula (1). The voltage divided by the current will reveal the impedance at each density. The graph of Fig. 8 shows the values obtained plotted against "B."

It is true that impedance is equal to the voltage divided by the current, but in view of the distorted shape of the current wave form we ought to think carefully how it should be measured. The true impedance to 50 c/s will be the voltage divided by the 50 c/s fundamental of the current. The sine curves in Fig. 3 were taken with a known current, so that the analysis of the distorted curves could be converted into actual current values. It was found that the impedances obtained by using the 50 c/s fundamental taken from the oscillograms were practically identical with those given when the distorted current was measured with a rectifier-type meter. This class of instrument gives a reading proportional to the *mean* current.

Returning to Fig. 8, the scale of impedance or inductance is an arbitrary one. The specific values in any particular case will depend upon the size of the core of the transformer and upon the number of turns on the primary winding. But whatever the design may be, the impedance or inductance will vary in the same manner as the flux density is changed, providing the transformer has a closed iron circuit of the same magnetic material i.e., Silcor 2. By taking a single test at any known flux density, the inductance at any other density can be obtained by reference to Fig. 8.

In Part II it will be shown how the information expressed in the graphs of Fig. 5 and Fig. 8 can be used to calculate the distortion produced by *any* push-pull transformer (providing it has a closed magnetic circuit of Silcor 2) when used in *any* specified circuit.

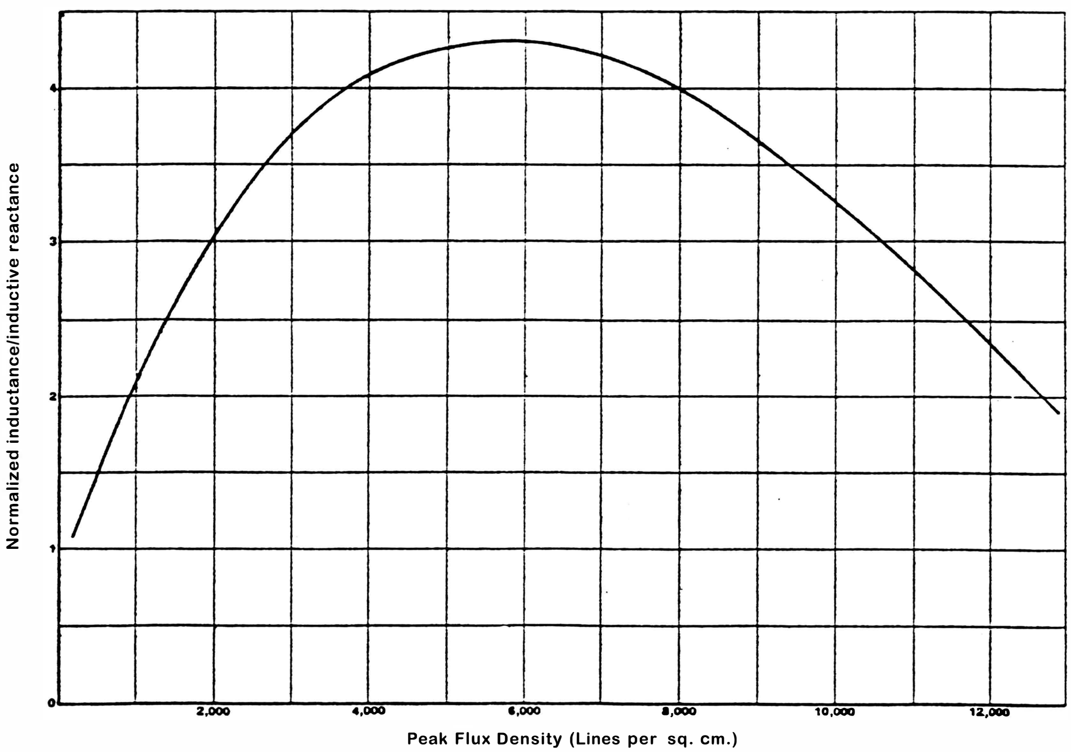


Fig. 8. Illustrating how the inductance/inductive reactance of a transformer with a closed core of, for instance, Silcor 2 vary with induced flux density. Inductance/inductive reactance is normalized.

APPENDIX

An analysis of the wave form illustrated in Fig. 7 gave 58 per cent. third harmonic, 34 per cent. fifth harmonic, and 39 per cent. seventh harmonic. These figures are very much higher than the corresponding percentages present in the distorted current curves (see Fig. 3 and Fig. 5). This is reasonable, because if the distorted current can be represented by:

$$I = A_1 \sin(\theta + \beta_1) - A_3 \sin(3\theta + \beta_3) - A_5 \sin(5\theta + \beta_5), \text{ etc.,}$$

one would expect distortion of approximately the same order in the distorted flux curve when a sine wave current is passed through the coil. But the induced voltage is proportional to the rate of change of flux—i.e., $\frac{d\phi}{dt}$, and the process of differentiation changes the ratio of the constants to A_1, $3A_3$, $5A_5$, etc., thus accentuating the harmonics.

No such simple relationship is found in practice. When the flux is distorted the iron losses increase, owing to the presence of third, fifth, and seventh harmonic eddy currents, etc. These losses may be represented as resistive loads across the primary of the transformer tending to reduce the apparent voltage distortion. Hence the accentuation of the higher harmonics in the voltage wave form is not so great as might be expected.

Distortion in Transformer Cores

Part II. THE "PARTRIDGE DISTORTION INDEX" AND ITS CALCULATIONS

By N. PARTRIDGE, B.Sc. (Eng.), A.M.I.E.E.

IN Part I of this series of articles the author showed that the iron core of any output transformer can give rise to serious harmonic distortion. Part II explains by a number of examples how the magnitude of this distortion may be calculated in practical cases.

IT can be shown both experimentally and theoretically (see appendix) that the harmonic distortion appearing in the voltage across the secondary terminals of an output transformer, having a closed magnetic circuit not polarised by DC, is:

$$\text{Distortion (per cent.)} = x \times \frac{R}{Z_F} \quad \ldots\ldots \quad (3)$$

where x is the percentage distortion taken from the curves of Fig. 5, R is the parallel impedance of the valve and its external anode load taken together, and Z_F is the impedance of the transformer primary to the fundamental frequency.

TABLE 1

B	Z_F	x from Fig. 5			$x \times \frac{R}{Z_F}$ (R = 2,500)			
		3rd	5th	7th	3rd	5th	7th	Total
2,000	16,000	11.0	5.3	4.5	1.72	0.83	0.70	3.25
4,000	21,800	17.2	9.5	7.0	1.97	1.08	0.80	3.85
6,000	22,900	22.3	13.1	10.6	2.43	1.43	1.16	5.02
8,000	21.200	27.0	16.0	15.6	3.18	1.88	1.84	6.90
10,000	17,300	31.0	18.7	20.3	4.48	2.71	2.94	10.13

The reader is advised not to worry too much about the origin of this formula. Mathematical reasoning is very fascinating, but must not be allowed to interrupt the major line of thought. Our immediate interest is centred upon the behaviour of output transformers, and formula (3) will yield the desired information providing it is used properly. Let us examine the various terms one at a time.

"R" is the combined impedance of the AC resistance of the valve and its effective anode load when connected in parallel. The former is a reasonably constant figure which can be extracted from the data given by the valve manufacturers. It depends upon the characteristics of the valve and the manner in which the valve is used, but is quite independent of the transformer design. The external anode load will be the true load connected across the secondary of the output transformer multiplied by the square of the transformer ratio. The anode load, as calculated in this manner, should normally be the optimum load for the valve.

Z_F and x are both variable, and are dependent upon the frequency and the flux density (B) in the core of the transformer. The way in which Z_F varies with flux density in the case of Silcor 2 has been illustrated in Fig. 8. If the iron losses are overlooked, the simplest method of determining Z_F is to measure the inductance of the transformer primary at any known flux density. The value of inductance at any other density can be deduced from the information contained in Fig. 8, and the impedance, to a sufficiently close approximation for the present purpose, is:

$$Z_F = 2\pi \times \text{frequency} \times \text{inductance} \quad \ldots. \quad (4)$$

The values of x can be taken directly from Fig. 5, providing the core material is Silcor 2. It can be the percentage content of any one harmonic at the chosen flux density or, alternatively, it may be the total harmonic distortion. It is, of course, absolutely essential that Z_F and x be evaluated at one and the same flux density.

Practical Examples

To examine the degree of distortion that may be found in practice, three different transformer designs will be reviewed. The calculations relating to the first example will be given in full so that the reader can see exactly how formula (3) is used in conjunction with Fig 5 and Fig. 8.

Example 1. Take an output stage consisting of two DO24 valves in Class "A" push-pull, giving a maximum output of 12 watts. The optimum load given by the makers is 5,000 ohms, anode to anode, and the AC resistance per valve is 2,500 ohms.

A transformer having a primary winding of 2,200 turns upon a 1in. stack of No. 4 stampings of Silcor 2 will have a measured impedance of approximately 21,800 ohms at 50 c/s when the flux density is 4,000 lines per sq. cm. Given this one value of the impedance at a stated flux density it is easy to deduce all the other values by reference to Fig. 8. Table 1 shows the impedance at five different values of B. It should be noted that the primary impedance varies between three and four times that of the optimum load and, therefore, the attenuation at 50 c/s will be very small.

TABLE 2

Frequency c/s	Watts	B	Z_F	x	$x \times \frac{R}{Z_F}$
40	12	12,900	8,180	110.0	33.6
50	12	10,300	16,700	71.0	10.6
70	12	7,350	30,900	54.5	4.4
90	12	5,720	41,300	44.3	2.68
110	12	4,680	49,500	37.8	1.90

Columns 3, 4 and 5 of Table 1 contain the values of x for the third, fifth and seventh harmonics respectively. These are taken directly from Fig. 5. Assuming the ratio of the transformer has been correctly chosen, the anode-to-anode load will be 5,000 ohms. The total valve impedance is also 5,000 ohms (2,500 × 2) and hence R becomes 2,500 ohms. This figure is constant throughout the ensuing

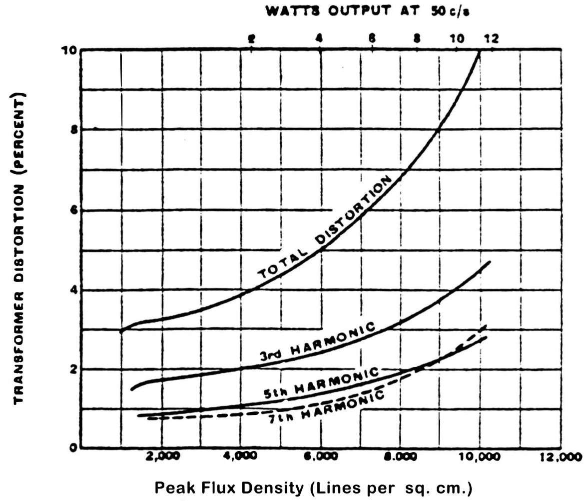


Fig. 9. The distortion produced at 50 c/s by the transformer described in the text above, for use with two DO24s in push-pull. Table 1 shows how the distortion figures are calculated.

calculations, but Z_F is different for each flux density.

Returning to Table 1, columns 6, 7 and 8 with the results of multiplying x by $\frac{R}{Z_F}$ while column 9 shows the total distortion, i.e., the sum of columns 6, 7 and 8. These figures have been plotted in Fig. 9 and the curves so produced tell the whole story concerning the distortion caused by the transformer at 50 c/s. By repeating the calculations similar sets of curves could be derived for any other frequency.

In addition to the flux density scale, a scale of watts has been added to Fig. 9. This is derived by using the formula:

$$\text{Volts} = \sqrt{\text{Anode Load} \times \text{Watts}} \quad .. \quad (5)$$

The voltage across the transformer terminals at any output can thereby be calculated and substituted in equation (1) to discover what value of B corresponds to the output in question.

Another very informative graph is shown in Fig. 10. The derivation of this curve may be traced from Table 2. It defines the total distortion that would be produced by the above transformer when delivering 12 watts at various frequencies. The total distortion has dropped to 2½ per cent. at 90 c/s. But below about 60 c/s the distortion increases very rapidly. This is due to the combined effect of *higher* flux density and *lower* impedance (Z_F).

The foregoing outline of the calculations relating to the problems of iron distortion have, perhaps, made this section of the article a little heavy. The reader is asked to forgive this, because the subject matter is quite new and a full explanation was therefore unavoidable. However, having shown the method attention can be turned to the lighter side, that of discussing the results. The transformer selected for Example 1 is obviously useless as far as high fidelity reproduction is concerned; somewhere around 10% distortion at 50 c/s is far more than can be tolerated. But the reader should observe with special care that the transformer would classed as very good if judged by normally accepted standards.

Taking the usual "selling points" one at a time we find:

(1) *Frequency Response:* This is excellent. The primary impedance is three to four times the load impedance at 50 c/s. (*The Wireless World* has often recommended a ratio of two as being adequate), hence the bass is well looked after. The high-frequency response depends only upon the method of winding the bobbin and can be taken as level up to 20,000 c/s for the sake of argument.

(2) *Ratio.* Our calculations have assumed that the ratio was exact.

(3) *Resistance of Windings.* The primary resistance would be in the region of 80 ohms total, which is commendably low for an optimum load of 5,000 ohms.

Defining Performance

The author has frequently made derogatory remarks concerning the present craze for lauding frequency response as a proof of excellence. And here we have a typical example of its very limited indications. The transformer examined above passes the much vaunted "straight line" test with flying colours, in spite of being in fact a very third-rate article.

Obviously, additional tests are indispensable. A statement of harmonic distortion is imperative for the purpose of comparing the merits of various output transformers. The author suggests that a very simple scheme would be to state *the total percentage distortion produced at 50 c/s when the transformer is delivering its*

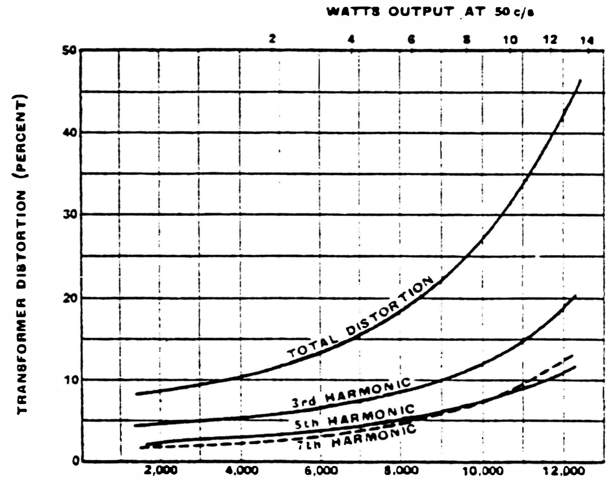


Fig. 11. High impedance valves such as pentodes and tetrodes accentuate the transformer's distortion generating proclivity. These curves apply to a transformer working in conjunction with push-pull KT66s.

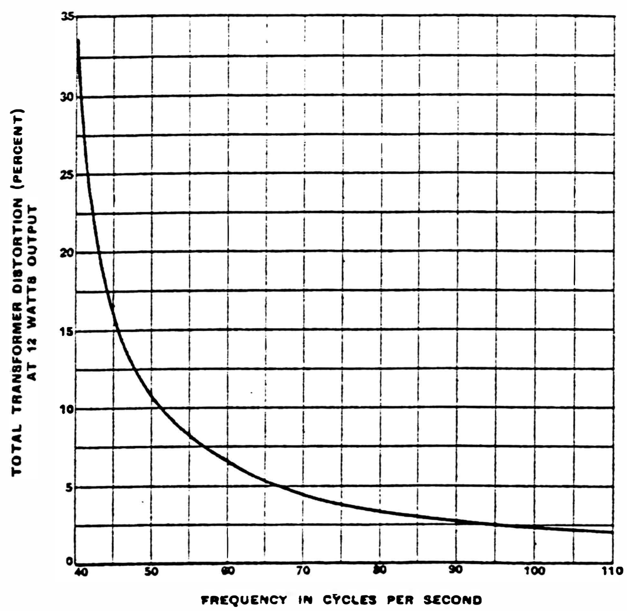
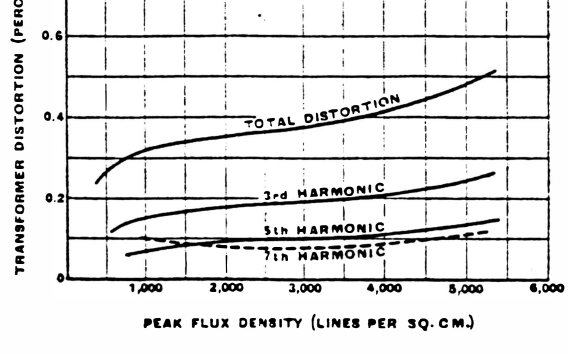


Fig. 10. Showing how the total distortion at full load varies with frequency. The origin of this curve is indicated in Table 2 while the transformer to which it applies is the same as that analyzed in Table 1.

TABLE 3

B	Z_F	x from Fig. 5			$x \times \frac{R}{Z_F}$ (R = 4,000 approx.)			
		3rd	5th	7th	3rd	5th	7th	Total
2,000	9,630	11.0	5.3	4.5	4.57	2.21	1.87	8.65
4,000	13,000	17.2	9.5	7.0	5.28	2.92	2.15	10.35
6,000	13,600	22.3	13.1	10.6	6.55	3.84	3.11	13.50
8,000	12,700	27.0	16.0	15.6	8.50	5.03	4.92	18.45
10,000	10,300	31.0	18.7	20.3	12.0	7.26	7.90	27.16
12,000	7,400	35.0	20.0	23.0	19.0	10.8	12.4	42.2

Fig. 12. By careful design, output transformer harmonic distortion can be reduced to very low levels. Below is the performance of a transformer working with two DA30 valves in push-pull.



full rated output into a resistive load, of value equal to the nominal secondary load. The corresponding figure for the above transformer would be 10.6 per cent. It must be remembered that such a distortion index is arbitrarily chosen and, in order to emphasise this and to avoid confusion with other possibilities, it will be referred to as the *"Partridge Distortion Index."*

Example 2. As a second example we will try two KT66 valves in push-pull, operating with an anode-to-anode load of 4,000 ohms. The AC resistance is very high (about 25,000 ohms each) and the output 17 watts.

Again keeping the No. 4 stampings and winding 1,700 turns upon a 1in. stack, the impedance will be as indicated in Table 3. The distortion curves are reproduced in Fig. 11, which corresponds to Fig. 9 of the previous example, and it can be seen that the Partridge Distortion Index is too high to be estimated. Note that once more the frequency response curve would proclaim the transformer as good.

It will be appreciated from this example that high impedance tetrodes and pentodes accentuate transformer distortion. With triodes the value of R is usually considerably less than that of the optimum load because of the effect of the AC resistance of the valve. In the present case R is approximately equal to the optimum load since the valve impedance is too large to have much effect.

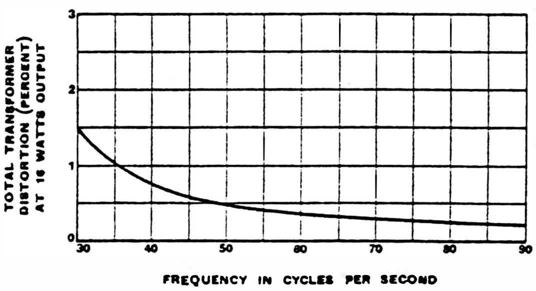
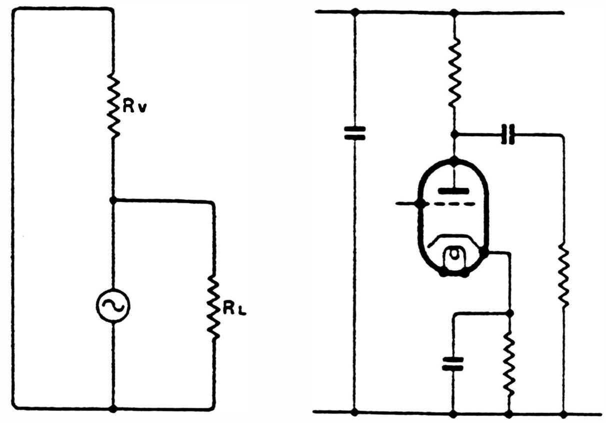


Fig. 13. These curves relate to the same transformer as Fig. 12 and show how the distortion at full load varies with frequency.

Example 3. As a final illustration which can be taken as representative of good design, a transformer for use with two DA30s will be described. These valves require an anode-to-anode load of 9,000 ohms in Class "A," and give a speech output of 17 watts.

The transformer uses a $1\frac{1}{2}$in. stack of No. 4 stampings and is wound with 4,400 turns on the primary. Fig. 12 shows the distortion at 50 c/s. In this case the Partridge Distortion Index is 0.5 per cent. which is very satisfactory. In Part IV of this series it will be shown how even better results are possible by using a different magnetic material and a modified design technique. Fig. 13 shows how the distortion at full output varies with frequency and corresponds to Fig. 10 relating to Example 1.

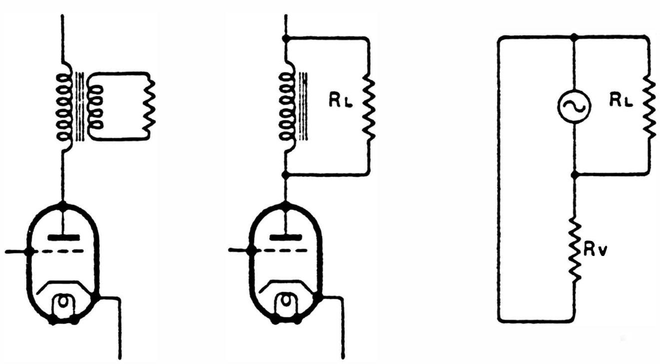
When reviewing the latter curve (Fig. 13) it must be borne in mind that the full output at 30 c/s can never be expected in normal use. If the full output is devoted to one frequency, nothing can be superimposed upon it without overloading the output stage. But music consists of many frequencies all reproduced at one and the same time. Hence the amplitude of the bass notes must be small compared with the maximum amplitude permissible. In the opinion of the writer the distortion on full load at 50 c/s can be taken as the maximum distortion likely to be encountered in practice.



(a) (b)

Fig. 15. At (a) is shown Fig. 14 (c) inverted. It is analogous to the usual valve circuit illustrated at (b).

As a parting reflection, how can the obvious superiority of the transformer described in Example 3 be detected by the tests usually applied to this type of component? Clearly it would be judged as simply another good transformer. What further proof is necessary to demonstrate



(a) (b) (c)

Fig. 14. Circuit (a) is the usual output transformer diagram which can be represented as at (b) by suitably choosing the value of RL. (c) is the equivalent of (b) if the transformer is considered as a generator.

APPENDIX

To prove that the percentage distortion produced by an output transformer is equal to $x \times \frac{R}{Z_F}$, consider the output transformer as a generator of harmonics. Fig. 14 (a) can be reduced to Fig. 14 (b), which is equivalent to Fig. 14 (c). Next, turn Fig. 14 (c) upside down. This has been done in Fig. 15 (a) and the similarity between this and the usual valve circuit of Fig. 15 (b) becomes at once apparent.

Falling back upon the well-known valve calculations, the harmonic voltage appearing across the load will be:

$$V_H \times \frac{\text{External Impedance}}{\text{Generator Impedance} + \text{External Impedance}} \quad \ldots (6)$$

where V_H is the open circuit harmonic voltage.

Let Z_F = primary impedance to fundamental frequency.

I_F = value of current at the fundamental frequency.

Z_H = primary impedance to any specified harmonic.

V_H = internally generated voltage of specified harmonic.

x = harmonic current produced by V_H expressed as a percentage of the fundamental current (I_F) when the external impedance is zero. Values of x were given in Fig. 5.

V_H = short circuit current $\times$ primary impedance.

$$= \frac{x}{100} I_F \times Z_H$$

Normal External Load $= \dfrac{1}{\frac{1}{R_L} + \frac{1}{R_V}} = R$ (say) (see Fig. 15 (a).)

Total impedance of the circuit $= R + Z_H$ (see Fig. 15 (a).) $= Z_H$ approx.

This approximation is justified in normal cases because Z_F is at least equal to R in any reasonably well-designed transformer, and Z_H will be several times Z_F since impedance increases almost in proportion to frequency. The addition of R and Z_H must be vectorially performed and is numerically approximately equal to $\sqrt{R^2 - Z_H^2}$ which is nearly equal to Z_H because Z_H^2 is large compared with R^2. Therefore:—

Harmonic voltage across load by substitution in equation (6)

$$= \frac{x}{100} I_F Z_H \times \frac{R}{Z_H} = \frac{x I_F R}{100}.$$

Fundamental voltage across load $= I_F Z_F$.

Therefore the harmonic voltage appearing across the load expressed as a percentage of the fundamental voltage becomes:

$$\frac{x I_F R / 100}{I_F Z_F} \times 100 = x \frac{R}{Z_F}$$

the immediate necessity for a statement of the Partridge Distortion Index or some other figure that will serve as a reliable guide? It is the only way the genuine article can be distinguished. A "straight line" response is not a passport to the land of high fidelity. The elimination of harmonic distortion serves a more useful purpose than the retention of frequency bands beyond the range employed for radio or gramophone.

Distortion in Transformer Cores

PART III—DC POLARISATION: INTERMODULATION EFFECTS: CHOICE OF CORE MATERIALS

By N. Partridge, B.Sc. (Eng.), A.M.I.E.E.

THE basic cause and nature of iron distortion in output transformers have been discussed in the two previous instalments. This article deals with the secondary effects that have to be considered before a reasonably complete understanding of the subject can be claimed.

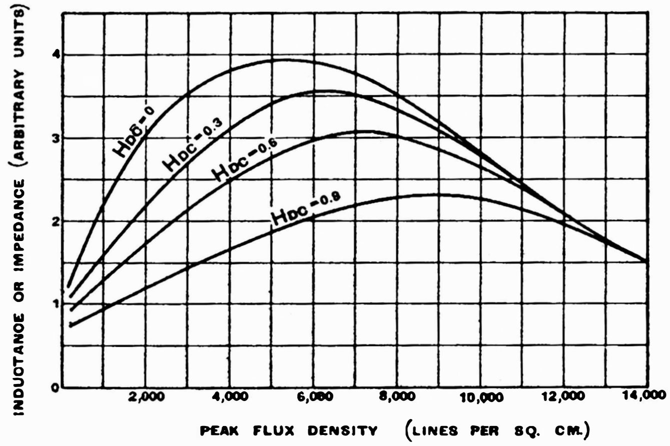


Fig. 17.—The relative inductances of a transformer with varying degrees of polarisation are shown for all values of B. The core material in this case was Silcor 2.

THE examples considered so far have all assumed a resistive load on the transformer and also that the anode currents of the push-pull valves were accurately balanced, so that the core was not polarised. In addition, only one grade of magnetic material has been examined analytically.

A loud speaker does not behave so conveniently as a resistance when constituting an output load. A resistance maintains the same ohmic value at all frequencies,

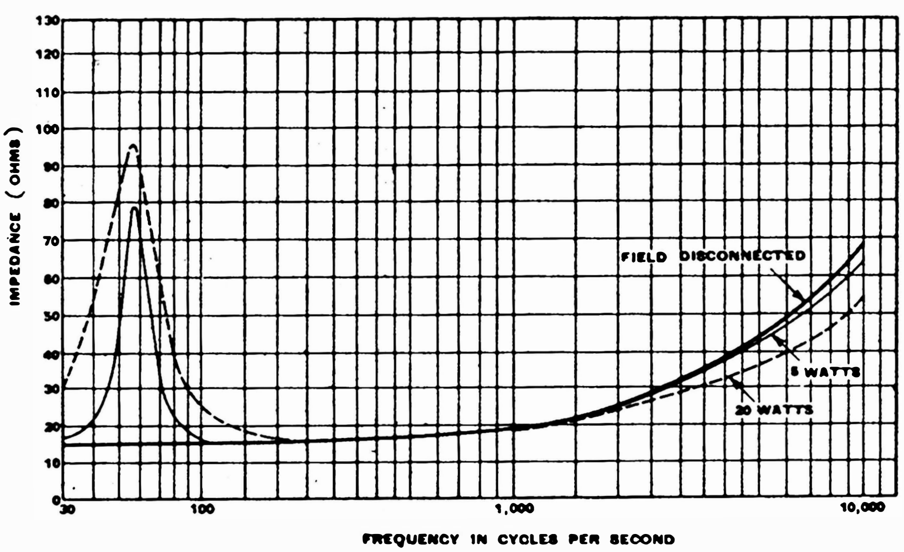


Fig. 16.—The impedance of a moving coil speaker varies considerably with frequency and the field strength. The rise around 50 to 60 c/s accentuates transformer distortion.

but a speaker varies enormously. Fig. 16 shows the impedance curves of the Celestion E55 speaker. These curves were given to the author by Messrs. Celestion, Ltd., and indicate the variation of speaker impedance with frequency and how this curve changes with the speaker field strength.

The sharp rise of impedance at around 50 to 60 c/s, which is common to all speakers, is serious. It causes the value of R in the formula (3) to increase and the transformer distortion will be accentuated as a result. An example on the lines of those given last week will illustrate the point.

Consider an output transformer for two KT66's in push-pull, having 2,800 turns on a 1¼in. stack of No. 4 stampings. This is a more lavish design than that given in Example 2 in Part II, and Table 4 shows that the resultant iron distortion is noticeably less when the anode to anode load is 4,000 ohms. Column 4 applies to this condition and states the total percentage distortion at 50 c/s. If the resistive load be replaced by the E55 speaker (or any other for that matter) the value of R will become much higher than 4,000 at 50 c/s. The AC resistance of the valves is far too high to help very much. R is almost the same as the speaker impedance multiplied by the square of the transformer ratio. Columns 5, 6 and 7 of Table 4 show the total distortion at 50 c/s, when R is equal to 8,000, 12,000 and 16,000 ohms respectively. Although the transformer appears passably good with a resistive load, considerable distortion will be produced in practice when using a speaker in conjunction with high-impedance valves.

It is well known that direct current passing through the primary of a transformer polarises the core and causes a drop in the inductance. One of the advantages of the push-pull arrangement is that the anode currents of the two valves traverse the transformer windings in opposite directions and cancel each other magnetically. But an exact balance of these currents is unlikely unless some special precautions have been taken to ensure it. An examination of the influence of the small out-of-balance currents likely to be met in practice is therefore of interest.

Fig. 8 in Part I showed how the inductance or impedance of a transformer varies with the AC flux density in the core. Fig. 17 repeats this curve together with

TABLE 4

B	Watts Output R=4,000	Z_F	Distortion (per cent.) R=4,000	R=8,000	R=12,000	R=16,000
1,000	0.5	28,000	1.85	3.7	5.5	7.4
2,000	2.0	39,000	2.1	4.2	6.3	8.2
3,000	4.6	48,000	2.3	4.6	6.8	9.2
4,000	8.1	52,500	2.5	5.0	7.5	10.0
5,000	12.7	55,000	2.9	5.8	8.8	11.6
6,000	18.0	55,000	3.3	6.6	10.0	13.2

three additional curves which show how drastically a polarising current reduces the impedance. The values of the DC magnetising force are marked on the curves and they correspond roughly to out-of-balance currents of 2.5, 5 and 10 mA in a transformer such as that analysed in Table 4. Obviously, the effect of even a small out-of-balance current is not negligible. Formula (3) states that the effective distortion will be increased in inverse proportion to Z_F. It follows that some means of securing equality of the

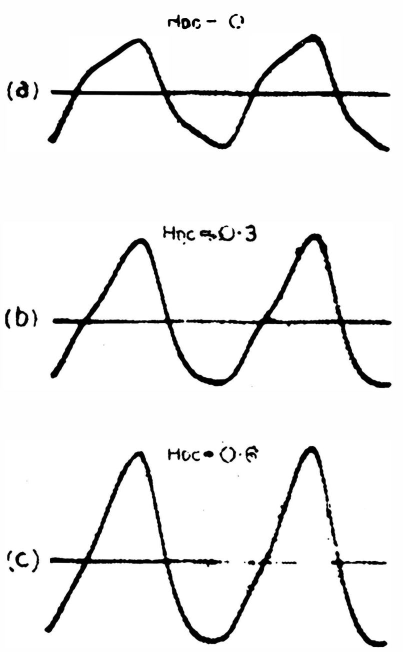


Fig. 18.—Small out-of-balance between the anode currents of push-pull valves is sufficient to reduce the transformer inductance and to add even harmonics to the existing iron distortion.

anode currents should be provided in high fidelity push-pull amplifiers.

Nor is the loss of inductance the only result of polarisation. Fig. 18 gives oscillograms showing the production of even harmonics. At (a) is the current wave form at an AC flux density of 2,920 lines per sq. cm. (no DC component). This oscillogram is the same as the corre-

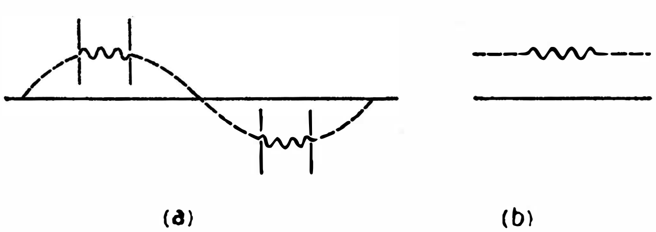


Fig. 19.—Showing the similarity between (a) a high frequency superimposed upon the peak of a low frequency and (b) superimposed upon DC.

sponding one in Fig. 3. but the current scale is smaller. Superimposing a DC magnetising force of H=0.3, roughly equivalent to 2.5 mA difference in the anode currents of two push-pull valves, resulted in the waveform at (b). The current is larger owing to the lowering of the inductance and also the wave form is not symmetrical about the zero line. The latter points to the presence of even harmonics. The third oscillogram (c) shows how both effects are magnified by doubling the DC field strength (H=0.6).

The preceding considerations relating to the effect of DC lead to an interesting line of thought. Imagine two widely different frequencies (DC excluded) being fed into

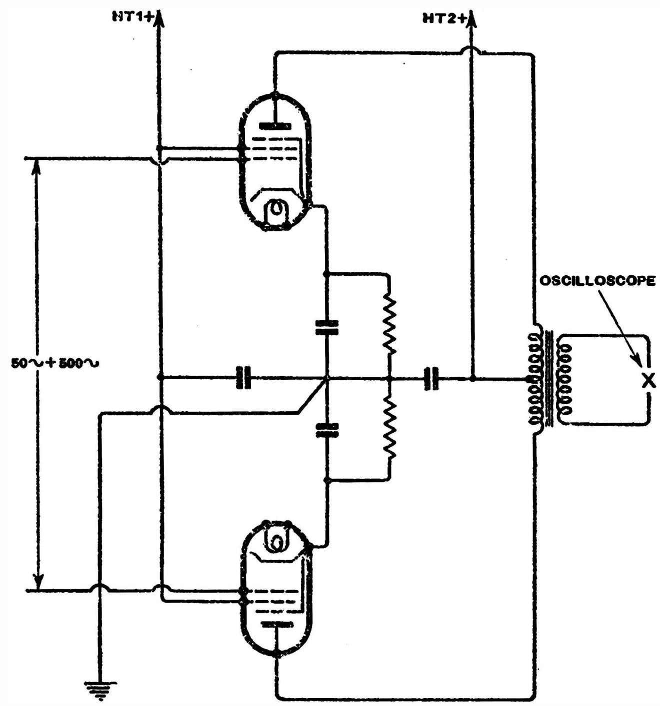


Fig. 20.—The type of circuit employed to obtain the oscillograms reproduced in Fig. 21.

a transformer simultaneously, as must often occur in a normal programme. Periodicities of 50 and 500 c/s would suit the case. How can the higher frequency distinguish between the peaks of the 50 c/s current and a direct current? Fig. 19 makes the similarity between the two conditions easily seen. If the current at the lower frequency behaves in the manner of DC when at its peak values, we should expect the higher frequency to be modulated by the lower frequency.

To show that such an effect does indeed take place, a circuit similar to that shown in Fig. 20 was set up. The important points to note about it are: (1) high impedance valves are used, and (2) the transformer is virtually unloaded since the oscillograph has a very high input impedance. The resultant oscillograms, therefore, show the wave form of the open circuit *voltage*, which exaggerates distortion to a maximum as was mentioned in the Appendix to Part I of the series.

Fig. 21 (a) shows the wave form of the 50 c/s output *voltage* when the flux density in the core of the transformer was of the order of 300 lines per sq. cm. At (b) is shown the wave form of the 500 c/s output. The flux density in this case was very small owing to the higher order of the frequency, and, therefore, little iron distortion can be observed. The next oscillogram (c) depicts the state of affairs when the two frequencies were applied together. Note that the 500 c/s wave is not simply added to the wave shown at (a), but it has been modulated as well. The variations of amplitude are

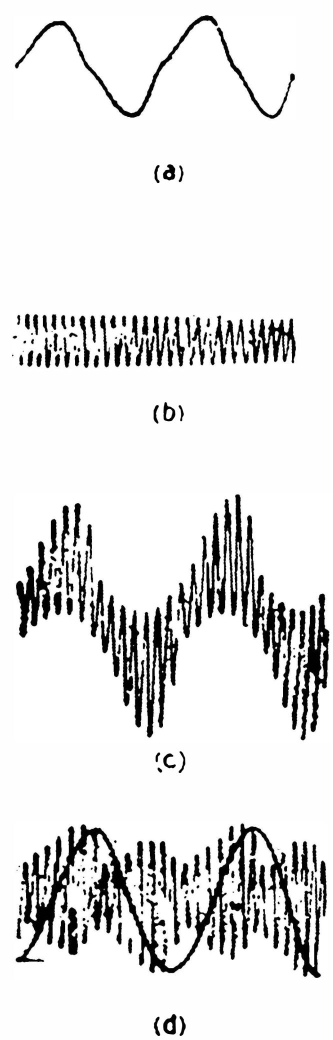


Fig. 21.—The magnetic characteristics of iron are such that high frequencies can be modulated by low frequencies. It is shown in the text that the effect is not very important in practice.

easily detected, but to make it still more obvious, a device was rigged up that allowed the low frequency voltage to be taken out, leaving only the distorted or modulated 500 c/s wave. This is given in

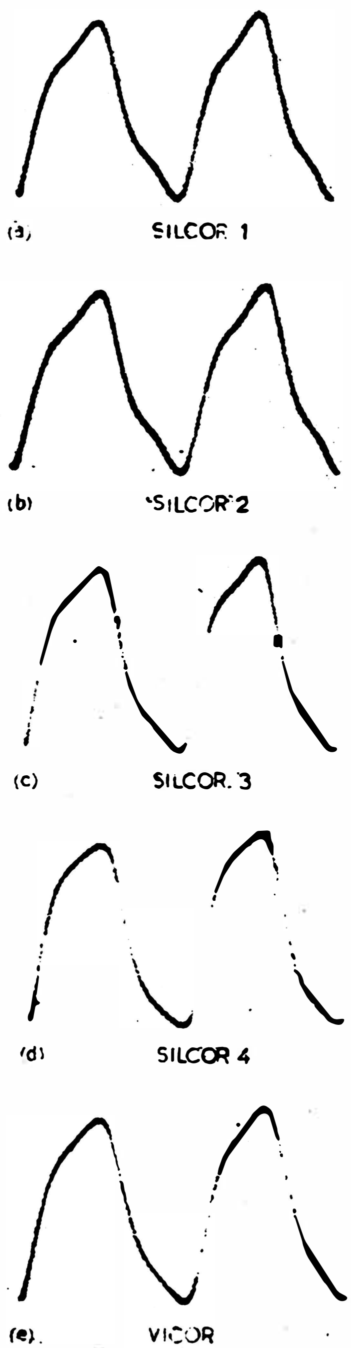


Fig. 22.—The current distortion varies widely with different magnetic materials. The above oscillograms were taken at a flux density of 4,680 lines per sq. cm.

Fig. 21 (d). The sine wave marks the phase relationship of the 50 c/s *current* through the primary, which was responsible for modulating the 500 c/s wave.

In reality the facts illustrated in Fig. 21 (c) and (d) are not quite so simply explained as it might appear from the above. But this is of no immediate consequence. The point is that high frequencies *are* modulated by relatively lower frequencies. Fortunately, several factors are present that prevent this type of distortion from being prominent in practice. The modulated wave can be looked upon as the original frequency plus a number of others superimposed. Using the radio analogy, the 500 c/s has acquired "side-bands." The transformer can again be accepted as the generator of the unwanted frequencies and a method of calculating the distortion in normal conditions can be deduced. Space does not allow of a full description, but it will suffice to say that the effect is negligible compared with that of the harmonic iron distortion to which this article is mainly devoted.

The laminations or stampings for transformers can be obtained in a number of different magnetic materials. All the results described up to this point have applied to the alloy known as Silcor 2, manufactured by Messrs. Magnetic and Electrical Alloys, Ltd., of Wembley. Other possible materials supplied by the same firm are Silcor 1, Silcor 3, Silcor 4, and a rather different alloy known as Vicor. The magnetic characteristics of each are different and it would be reasonable to anticipate variations in the degree of distortion caused by these alternatives.

To investigate this matter, current oscillograms were taken for each material at a

TABLE 5

Material	Percentage Harmonic Distortion (Current). B = 4,680			
	3rd	5th	7th	Total
Silcor 1 ...	17.3	9.3	10.9	37.5
Silcor 2 ...	20.0	11.1	8.6	39.7
Silcor 3 ...	18.2	9.2	6.3	33.7
Silcor 4 ...	15.6	7.8	3.2	26.6
Vicor ...	14.9	8.0	5.6	28.5

flux density of 4,680 lines per sq. cm. The photographs are reproduced in Fig. 22 and the results of the harmonic analyses are given in Table 5. These oscillograms and the distortion figures obtained from them may be compared with Fig. 3 and the point on the curve of Fig. 5 corresponding to B=4,680. The conditions of test are the same, but the core material is varied. One cannot jump to an immediate conclusion as to the relative merits of these materials merely by consulting Table 5. The basic distortion is not so important as the actual distortion under normal conditions of use and formula (3) told us that this depends upon $x \times \frac{R}{Z_F}$.

R is a constant of the circuit, but x and Z_F depend upon the characteristics of the iron. The values of x are to be found in Table 5 for B=4,680, but Z_F is not disclosed.

TABLE 6

Material	Z_F	Total Distortion (per cent.)	$\frac{x}{Z_F}$
	B = 4,680		
Silcor 1 ...	460	37.5	.0086
Silcor 2 ...	420	39.7	.0100
Silcor 3 ...	340	33.7	.0105
Silcor 4 ...	265	26.6	.0107
Vicor ...	490	28.5	.0061

A curve similar to that given in Fig. 8 is required for each material. These will be found in Fig. 23. Using the latter curves in conjunction with Table 5 some idea of the merits of the several alloys can be obtained and also some information on how they should be used to the best advantage.

First of all, we will consider the result of substituting one in place of another without altering the windings or core area of the output transformer. Obviously, such a proceeding would change the values of both x and Z_F in formular (3) and the excellence of any particular core will depend upon the ratio $\frac{x}{Z_F}$. In Table 6 the total distortion produced by each grade of iron has been set out, together with the relative values of Z_F extracted from Fig. 23. The final column shows the relative distortion figures $\left(\frac{x}{Z_F}\right)$.

To make the significance of this quite clear, an example will be given. Suppose an output transformer has a core of Silcor 2, such as any of those described earlier in the article. If the iron be removed and, say, Silcor 4 substituted, two major changes in the characteristics of the transformer will be brought about. First, the

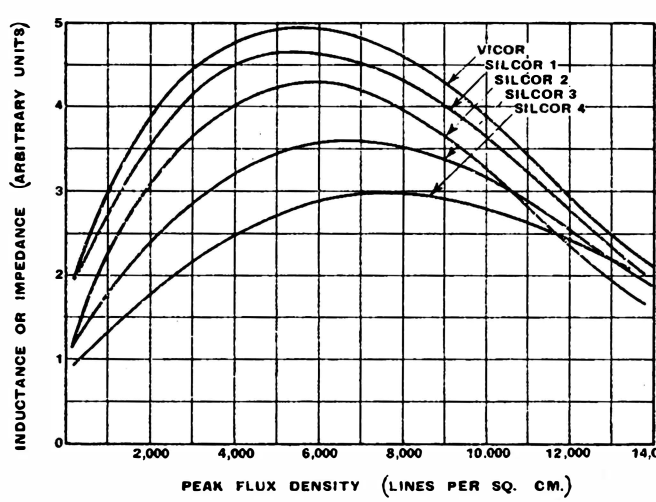


Fig. 23.—This graph illustrates the relative inductances that would be obtained by substituting five different grades of iron in a transformer.

inductance of the primary will be reduced in the ratio of 420 to 265 (see Table 6) and secondly, the percentage distortion caused by the iron at a flux density of B=4,680 will be increased to 1.07 times its original value. Note carefully that the *iron* distortion will be only slightly increased, but it is possible under certain circumstances that the loss of inductance may considerably increase *valve* distortion owing to the anode load falling below its optimum value.

Looking at each of the samples mentioned in Table 6 in a similar way to that just described, it will be noted that Vicor is outstandingly good. The concluding part of this series will deal with the application of this material to ultra-high fidelity output transformers.

Instead of substituting one type of core for another, the design of the entire transformer might be modified to accommodate the new material. For example, suppose we have a well-designed output transformer with a core of Silcor 4 and, being attracted by the high permeability of Silcor 1, we decide to employ this alloy for the production of an *electrically* similar transformer. The substitution of Silcor 1 for Silcor 4 would increase the inductance from 265 to 460 (see Table 6). Hence one would be justified in reducing the core area in the same proportion in order to end up with the same inductance as the original transformer, which we assumed was adequate. The usual frequency response test would show the new, smaller transformer to be as good as the original large one. But the Partridge Distortion Index would tell a different tale. Since Z_F has been made the same for both transformers, the figure of merit given in Table 6 no longer applies, and the ratio of the basic distortion figures (26.6 per cent. and 37.5 per cent.) must be used. Actually, the position is very much worse than this because by reducing the core area the flux density has been correspondingly increased and, therefore, a much higher distortion figure must be taken for the Silcor 1.

These illustrations show that the substitution of a higher grade of core material results in a small improvement in the iron distortion produced by a transformer. But to employ a high grade for the purpose of reducing size and/or weight results in a substantial increase of distortion. It would seem that a good output transformer *must* be large. Also, a large transformer using low-grade iron will give rise to less harmonic distortion than a small transformer having the same inductance but using high-grade iron.

In fact, the terms "high grade" and "low grade" are not at all applicable as far as speech transformers are concerned. The terms originated with reference to mains transformers, where core losses are so very important and one must guard against wrongly imagining that what is good for a mains transformer is necessarily good for a speech transformer.

Distortion in Transformer Cores

Part IV—REVISED DESIGN TECHNIQUE TO MINIMISE HARMONIC DISTORTION

By N. Partridge, B.Sc. (Eng.) A.M.I.E.E.

THE nature and extent of harmonic distortion in push-pull output transformers has been examined in detail in earlier instalments. This article, the last of the series, will be devoted to the consideration of ways and means of keeping this distortion under control.

IN designing a transformer for low distortion the first step is to select a "good" magnetic material for the core. In last week's instalment reasons were given for accepting Vicor (manufactured by Magnetic and Electrical Alloys, Ltd., of Wembley) as our starting point. An oscillogram showing the current distortion produced by this alloy at a flux density of 4,680 lines per sq. cm. was reproduced in Fig. 22, but to perform detailed calculations the distortion at all densities must be known. A series of current oscillograms at various flux densities is given in Fig. 24 and the graph obtained by analysing these wave forms is shown in Fig. 25. These two illustrations correspond to Fig. 3 and Fig. 5, which give the same information about Silcor 2. The final requirement is a curve connecting inductance with flux density. Such a curve is contained in Fig. 23, but for completeness it is reproduced here in Fig. 26.

Having fixed upon the core material, the second step is to consider how best it may be used. One could design a transformer in the conventional manner and claim an improvement by virtue of the better core. But there would still be one or two rather disconcerting criticisms. For one thing, the *intrinsic* distortion would be high. As can be seen from Fig.

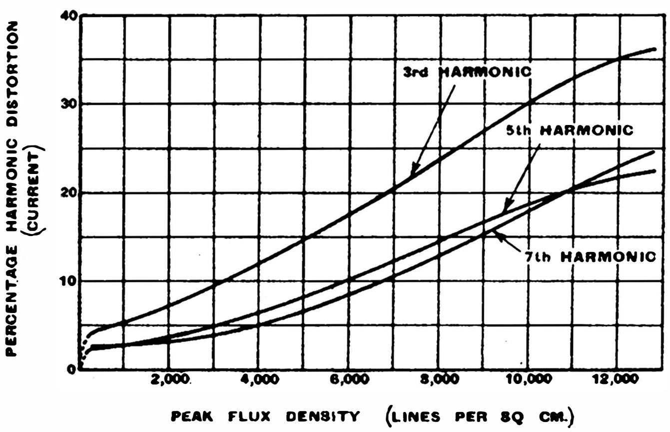


Fig. 25.—The graph obtained by plotting the result of an harmonic analysis of the wave forms of Fig. 24. The harmonics are expressed as a percentage of the fundamental.

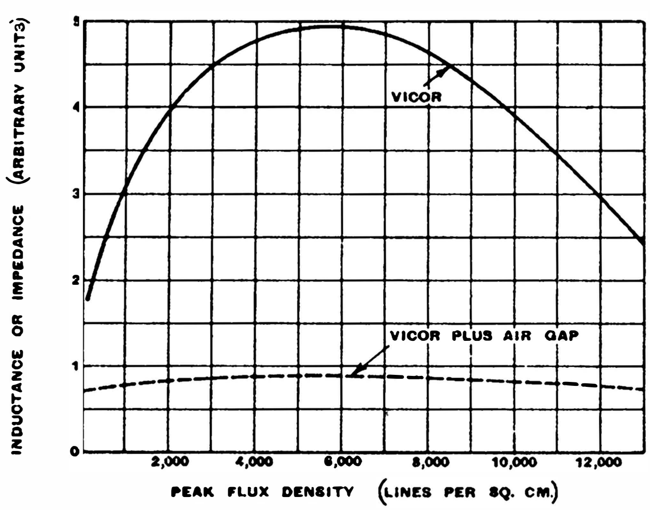


Fig. 26.—The full curve indicates the change of inductance (or impedance) with flux density in the case of a transformer having a closed magnetic circuit of Vicor. The dotted curve applies to a composite core of Vicor plus an air gap (see Table 7).

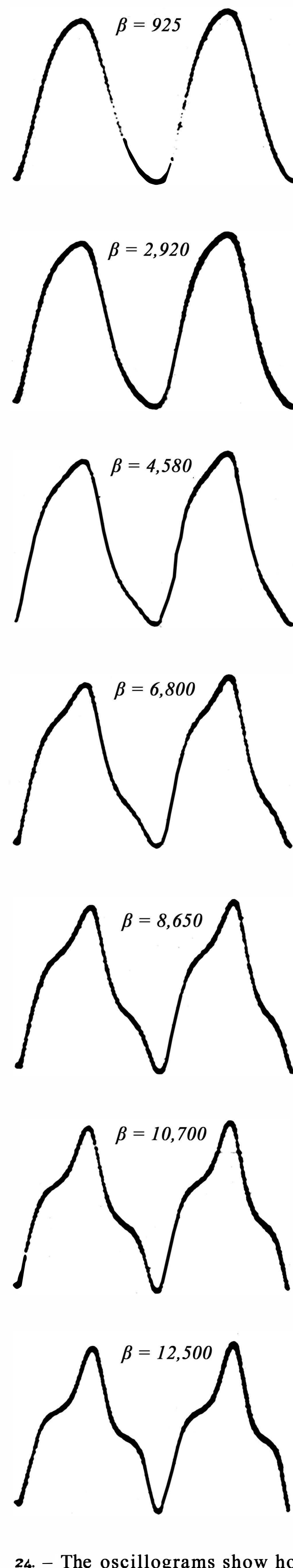


Fig. 24.—The oscillograms show how the current distortion varies with flux density in the case of Vicor. "B" is the value of the peak flux density in lines per sq. cm. The photographs should be compared with Fig. 3 (Part 1), which gave the same information about Silcor 2.

25, appreciable distortion occurs at quite low densities and it is only the somewhat fortuitous circuit conditions $\left(\frac{R}{Z_F}\right)$ that keep the working distortion within reasonable limits. It would be more satisfying if the transformer in itself could be made distortionless apart from the external circuit. Again, a small out-of-balance between the anode currents of the two push-pull valves will be sufficient to upset all the calculations. And there is still the little matter of frequency modulation which depends upon the external circuit for correction.

There is an extremely simple device whereby most of the troubles and worries mentioned above can be substantially lessened. That is by putting a suitable air gap in the magnetic circuit. Gaps have always been used for chokes and transformers carrying DC, but as far as the author is aware, such a technique has not been deliberately used by manufacturers to reduce intrinsic distortion apart from the question of polarisation.

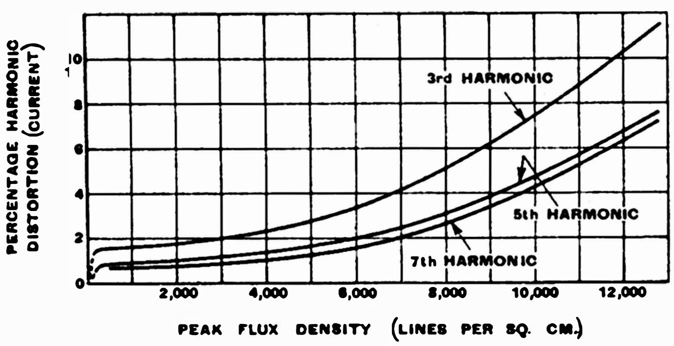


Fig. 27.—The intrinsic distortion of the core is materially reduced by an air gap. This graph should be compared with Fig. 25 which shows the distortion without a gap.

The effect of a gap can be easily understood with the aid of Fig. 26 and Table 7. Suppose a transformer, giving the relative inductances shown in the graph (Fig. 26) has a gap made in its core of a length such that the inductance at B = 1,000 is reduced from 3 to, say, 0.73. These figures are, of course, purely relative, and the actual inductances may be anything, depending upon the core area and the number of turns on the primary. Since the impedance has been reduced, a greater magnetising current will flow. But the iron circuit still requires exactly the same current to magnetise it and to supply the various losses, from which it follows that the additional current must be that required to maintain the flux in the air gap. This additional current will be undistorted and will vary *directly* as the flux density.

Table 7 shows an approximate method of estimating the inductance and distortion curves for the composite core consisting of Vicor plus the air gap. Column 1 contains selected flux densities for which the relative magnetising currents taken by the Vicor are shown in column 2. These figures were obtained by testing the Vicor without a gap. The third column indicates the magnetising current required by the air gap, which is proportional to the flux density. The total current is tabulated in column 5, from which the new relative impedances can be deduced. It must be remembered that this method is only approximate because the magnetising currents for the Vicor and the air path are assumed to be in phase, and this is not strictly true.

The new impedance curve is drawn dotted in Fig. 26. The inductance has been greatly reduced by the gap but this is not necessarily important. The earlier examples have shown that any good output transformer has a far higher inductance than is strictly required for the preservation of the bass. Our new curve at least approximates to a straight line. In other words, instead of having an inductance that varies enormously with the signal voltage, we now have an inductance that remains sensibly constant. Another important advantage is that any normal out-of-balance between the anode currents will be far too small to have any effect upon the Vicor, which is protected in this respect by the gap.

Turning to the question of harmonic distortion, a change has occurred here, too. The curves in Fig. 25 give the distortion as a percentage of the fundamental. The air gap has increased the fundamental without altering the magnitude of the harmonic currents, and, therefore, these harmonics will be noticeably smaller when expressed as a percentage of the augmented fundamental. Columns 6, 7 and 8 show the revised distortion figures in the case of the particular gap chosen for the purpose of Table 7. These values have been plotted in Fig. 27.

Because the basic distortion (x) has been reduced to less than one-third of its original value it must not be assumed that

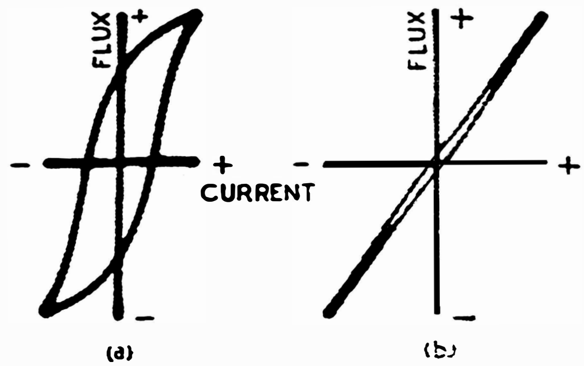


Fig. 28.—(a) shows the relationship between the instantaneous flux density and current in a closed core of Vicor. This approximates to the hysteresis loop. (b) gives the same information in the case of a gapped core. Note that the flux is almost proportional to the current.

a corresponding improvement will be found in the performance of the transformer. Actually, the working distortion has not been altered at all. Unfortunately, Z_F has been reduced just as much as x and the final result remains the same. But what we have done is to reduce the *intrinsic* distortion and make the performance of the transformer less dependent upon the external circuit. This modification is strongly reflected in the curve showing the relationship between the flux in the core and the magnetising current. Fig. 28 (a) shows this curve, which approximates to the hysteresis loop, for the ungapped transformer and Fig. 28 (b) repeats the curve for the gapped core. The latter is brought very close to the ideal, which would be a straight line.

The reader may be wondering why the gap chosen was one which reduced the inductance at B = 1,000 in the ratio of 3 to 0.73. At first sight it looks as though a much larger gap would still further reduce the intrinsic distortion and make the transformer behave as though it were air cored. This reasoning is perfectly correct, but there are practical limitations to the possible magnitude of the gap. The larger the gap the lower the inductance, and hence more turns have to be wound upon the primary in order to keep the inductance up to the minimum allowable value. Increasing the turns means using finer wire

TABLE 7

Peak Flux Density	Vicor Magnetising Current	Air Gap Magnetising Current	Total Magnetising Current	Impedance of the Gapped Core	Distortion of Gapped Core (per cent.)		
					3rd Harmonic	5th Harmonic	7th Harmonic
263	4.2	7.8	12.0	71.0	—	—	—
537	7.0	15.8	22.8	75.7	—	—	—
925	10.5	27.4	37.9	79.0	1.5	0.8	0.6
2,920	22.0	86.3	108.3	87.0	1.84	0.95	0.71
4,680	33.2	138.0	171.0	88.0	2.65	1.50	1.17
6,800	48.2	201.0	249.0	88.0	3.87	2.37	1.94
8,650	69.8	255.0	325.0	86.0	5.62	3.50	3.10
10,700	107.0	317.0	424.0	82.0	8.15	5.10	4.95
12,600	168.0	373.0	541.0	75.0	11.2	7.4	6.9

and obviously the wire gauge cannot be smaller than that which will safely carry the current. Also the DC resistance of the winding must not be permitted to reach too high a value.[1] Again, the leakage inductance must be kept within manageable proportions, and this limits the number of turns that can be employed.

With a view to showing the type of result that can be obtained with Vicor and the gap technique, a transformer was designed on a $1\frac{1}{2}$in. stack of No. 4 stampings to operate with two DA30 valves in Class A push-pull. The harmonic distortion given by this transformer at 50 c/s is indicated in Fig. 29. This should be compared with Fig. 12, which gives similar data relating to a well-designed transformer with a core of Silcor 2. Note that the Partridge Distortion Index[2] for the latter was 0.5 per cent., whereas the gapped Vicor reduces this figure to 0.2 per cent.

All the examples so far have employed the No. 4 stamping. The reason for this is that it is a very popular stamping and serves for the purpose of illustration as well as any other. But the No. 4 laminations are not necessarily the most suitable ones for audio-frequency transformer design. Greater iron section would be an advantage and so would be a slightly restricted window space. The former makes it possible to work at a lower flux density and the latter aids in the reduction of leakage inductance. The No. 56 stamping (Magnetic & Electrical Alloys, Ltd.) is a very good one. The dimensions of both the No. 4 and the No. 56 stampings are shown side by side in Fig. 30 for comparison.

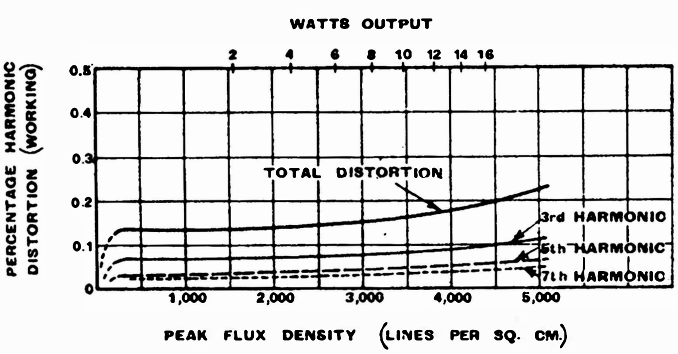


Fig. 29.—The distortion obtained under working conditions from an output transformer using a gapped Vicor magnetic circuit. The Partridge Distortion Index is only 0.2 per cent., which is very low considering the size of the component.

If expense is no great objection, the size of the transformer can be increased and, theoretically, the iron distortion can be reduced to as low a value as one wishes. A large core section with ample window space will permit the winding of a primary with a very high inductance and a large air gap will be possible without jeopardising the bass response. The intrinsic distortion will, by this means, be made extremely low, and, no matter what the external circuit conditions, such a transformer would not produce harmonic distortion. But theory and practice do not collaborate harmoniously in this respect. The larger the transformer the more difficult it becomes to preserve the high-frequency response. Also, owing to the shape of the distortion curves in Fig. 5 and Fig. 25, it requires a very considerable reduction in the flux density to bring about any worth while improvement in the transformer distortion.

The design of a good output transformer is beset with conflicting desiderata. The final solution must be a compromise and the best design is that which gives a well-considered balance of evils. The unpleasantness resulting from the loss of top, the iron distortion, etc., should all be approximately equal as judged by the ear. A superb frequency response is of no avail if harmonic distortion is high; a distortionless core is wasted if all the high frequencies are attenuated. To achieve such a balance requires not only technical knowledge but a wide practical experience as well.

Conclusion

Looking back upon the information brought to light by these investigations, perhaps the most striking thing is the fact that the articles should have been written at all so late in the development of electro-acoustics. Amplifier technique has been subjected to the most rigorous analysis in the cause of fidelity, and has long since reached a very high standard. Speech transformers were used in communication work years before radio was invented and yet, apart from vague apprehensions, nobody seems to have seriously worried very much about the extent of the harmonic distortion they produce.

As far as so-called commercial reproduction goes, iron distortion is not very important. It occurs only at l o w frequencies, and if true bass is not catered for in the amplifier, then it can do no harm in the transformer. But the subject must be studied with the utmost seriousness by those seeking really high-quality reproduction. Distortion at low frequencies is more dangerous than perhaps the reader has, as yet, appreciated. The characteristics of the ear are such that the sensitivity increases very rapidly from the lowest audible notes up to around 500 or 600 c/s. The effect of this is that *2 per cent. seventh harmonic contained in a 50 c/s note can sound as loud as the fundamental itself.*

This statement is truly amazing, but a few figures will prove its validity. A distortion of 2 per cent. means that the voltage of the seventh harmonic (350 c/s) is 2 per cent. of that of the fundamental (50 c/s). In other words, the seventh harmonic is 34 db below the level of the fundamental. But at a loudness level of 20 db the sensitivity of the ear increases by approximately 34 db between 50 c/s and 350 c/s. Hence the harmonic will sound to the ear as though it were 100 per cent.! One is, of course, assuming that the sensitivity of the loud speaker is the same at both frequencies. If it happens to be greater at 350 c.p.s., then the position is even worse.

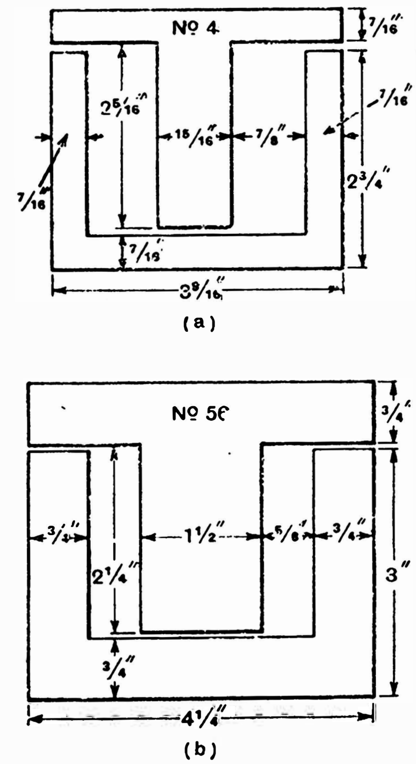


Fig. 30.- All the examples in this series have used the No. 4 stamping but this is not necessarily the best one. The No. 56 presents certain advantages mentioned in the text. The numbers are those of Messrs. Magnetic and Electrical Alloys, Ltd.

Obviously, something must be done about iron distortion. A transformer response curve is only a snare and a delusion when examined alone. The response is important up to a point, but it *must* be considered in conjunction with the transformer harmonic distortion. To do this a simple and standardised method of expressing the distortion is required, and the Partridge Distortion Index is put forward as a tentative suggestion. It may be defined as the arithmetical sum (*not* RMS) of the percentages of the third, fifth, and seventh harmonics produced under working conditions at 50 c/s when the transformer is delivering its full rated output into a resistive load of value equal to the nominal secondary load. By substituting a resistance in series with the primary to take the place of the valve AC resistance, the test can be taken using the 50 c/s mains as the source of power. This scheme eliminates all possibility of valve distortion masking the transformer distortion, and avoids the risk of polarisation.

[1] See "Output Transformers—The Effect of Resistance," *Wireless World*, January 12th, 1939.
[2] See Part II, June 29th issue and also last paragraph of this article.

Letters to the Editor

The Editor does not necessarily endorse the opinions of his correspondents

"*Distortion in Transformer Cores*"

IN reading the first instalment of Mr. Partridge's article in the June 22nd issue of *The Wireless World*, it occurs to me that one section might cause some confusion in the mind of anyone not very familiar with the subject. On page 573, column three, the author says "Note that both voltage and current have become distorted. This is to be expected, because the transformer draws a distorted current, and therefore the voltage drop across the series impedance must of necessity be distorted."

That is clear enough, but he goes on to say, "Hence the voltage across the transformer, which is the mains voltage *minus the distorted drop across the series impedance*, must also be distorted."

One would, I think, assume from this that the more distorted the drop across the series impedance, the more distorted would be the voltage across the transformer, whereas the reverse is true under the conditions stated.

Proof of the latter statement is provided by the extreme case referred to by the author in which the series impedance is very high compared with that of the transformer. He says "The current becomes a pure sine wave, and the distortion is transferred to the voltage curve." But his preceding remarks would lead one to argue that if the current becomes a pure sine wave, thus producing a pure sine wave voltage across the series impedance, then the transformer voltage, which is stated to be the mains voltage minus the drop across the series impedance, should also be a pure sine wave. Or, conversely, that if the transformer voltage is distorted the voltage across the series impedance must be distorted.

Actually, of course, assuming sine wave mains voltage, the current does not become a perfect sine wave, so long as an iron-cored impedance is in the circuit, although it may approximate to one. Harmonics must, therefore be present in the voltage across the series impedance, but the *percentage* is so small that the wave appears to be a true sine wave.

If we assume a true sine wave of current, then the transformer voltage harmonics must be present in the applied voltage. The voltage across the series impedance would then be a true sine wave.

I think it should also be made clear that the subtraction of the voltage across the series impedance from the mains voltage to give the transformer voltage, refers to instantaneous values, or the subtraction of one *wave* from another, as so many people are used to thinking in terms of RMS values.

The article in question, and any that follow on the same subject, should be most useful in emphasising the importance of a fundamental cause of distortion that receives far too little attention.

T. A. LEDWARD.
Huyton, Nr. Liverpool.

The Author's Reply

IT is perhaps possible that some readers may find occasions to become a little confused in the course of reading the articles in question. This is unfortunate but arises from the necessity of compressing a book full of information within the limits of four brief instalments. To do this one must present short and simple explanations, and assume that the technical man will amplify the theme for himself . . . as, indeed, Mr. Ledward has done.

My statements are all accurate and do not in any way disagree with those of Mr. Ledward. The voltage across the transformer *is* the mains voltage minus the distorted drop across the series impedance. When the series impedance becomes infinitely great compared with the transformer impedance, the current distortion becomes infinitely small and the current wave form can be as pure a sine wave as we care to imagine it. But in these circumstances, the drop across the series impedance is almost the same as the mains voltage itself. Hence subtracting one from the other leaves only a minute fundamental plus the said infinitely small distortion. But these two "almost nothings" are comparable, and hence a large *percentage* distortion appears in the answer. In other words, a pure sine wave minus a wave of equal magnitude containing an infinitely small harmonic content will leave only the said harmonic content which, although infinitely small, is nevertheless 100 per cent. of things its own size, so to speak.

I think that Mr. Ledward's letter in conjunction with my own somewhat sketchy observations will clarify the point in question and materially help readers who have found it to be a stumbling block.

N. PARTRIDGE.
London, S.W.1.

Letters to the Editor

The Editor does not necessarily endorse the opinions of his correspondents

"Distortion in Transformer Cores"

NOW that the series of articles by Mr. Partridge on "Distortion in Transformer Cores" is completed, I feel I must add to the letters of commendation that you and he are no doubt receiving. It is a notable contribution to the art of good reproduction. The amount of painstaking research and the data presented must arouse the admiration of all readers. I hope it will help to deflect a proportion of attention from frequency to amplitude characteristics. Incidentally, I agree with Mr. Owen Harries[1] in preferring the term *amplitude distortion* to *harmonic distortion*, because I am unable to accept Mr. Partridge's summary dismissal of intermodulation products—at least until he brings forward evidence more convincing than that of Mr. Harries[2] and "Cathode Ray's" experiment.[3]

This raises the vexed question of how to establish a criterion of amplitude distortion. Mr. Harries has suggested one in the article referred to. Messrs. Callendar and Clarke employ another,[4] based on weighting harmonics according to known physiologico-acoustical data. The R.M.A. has prescribed another, called the *distortion factor*, employing a less scientific but simpler weighting of harmonics, to recognise in some degree the relatively greater offensiveness of those of high order. *Total harmonic distortion* is another term with a definite accepted meaning, being the RMS sum of all the separate harmonics. It is a pity, therefore, that Mr. Partridge should have used this term in quite a different sense, because it is liable to cause confusion in an already complicated subject.

The "Partridge Distortion Index" applies only to output transformers under certain specified conditions, and so is not competitive with various existing standards of amplitude distortion in general. Mr. Partridge, therefore, has a perfect right to define it as it seems to him to serve the purpose best, and in view of his intense experience of transformer design I hesitate to question the reasons underlying his choice, yet I wish he could explain why he departs from the usual practice by adopting an arithmetical sum rather than RMS. And, more particularly, why he should attach no more weight to the 7th harmonic than to the 3rd, seeing that he is fully aware of its much greater offensiveness.

And that leads me to say that although I agree with him in the emphasis given to the relatively greater effect of the higher harmonics, the figures he gives to prove the point are chosen a little unfortunately for the purpose. In the first place, "a loudness level of 20 db" is a contradiction in terms: *intensity* is measured in db; but to avoid hopeless confusion it is absolutely imperative that loudness should be stated in *phons*. The context shows that phons are meant. But 20 phons is an altogether unpractical loudness for sound reproduction, being below the background noise of even quite a quiet room.

Moderately quiet reproduction is around 70 phons, and at that level the 34 db. difference in intensity between 50 and 350 cycles has diminished to 10 db. Moreover, the 50-c/s fundamental is more likely to mask the 350-c/s harmonic than *vice versa*.

These comments on certain details do not in any way intend to distract attention from the value of the series of articles as a whole, outstanding.

M. G. SCROGGIE.
Bromley, Kent.

[1] *The Wireless World*, July 21st, 1938.
[2] *The Wireless Engineer*, February, 1937.
[3] *The Wireless World*, May 19th, 1938.
[4] *The Wireless World*, August 25th, 1938.

Letters to the Editor

The Editor does not necessarily endorse the opinions of his correspondents

"*Distortion in Transformer Cores*"

BEFORE replying to the technical points raised by Mr. Scroggie in his letter of last week, I should like to express my appreciation of the exceedingly nice things he said about my articles on "Distorton in Transformer Cores." And at the same time may I be allowed the space to thank the many other readers who have written to me privately about the same matter?

Mr. Scroggie's first point relates to amplitude distortion and intermodulation products. The reason for my "summary dismissal" of this matter is that it did not seem quite so important in the case of a transformer as in that of a valve (for example) because of frequency discrimination in the former. Only spurious frequencies of a low order can find their way to the external load. However, it is quite possible that I have under-estimated this evil.

The next item, which deals with my use of the *arithmetical* sum of the harmonics, is very important. The "Partridge Distortion Index" is intended to fulfil two purposes: (1) to provide a simple means whereby professional engineers and amateurs alike can make reliable comparisons between transformers, and (2) to provide the transformer designer (as distinct from the circuit designer or transformer user) with a convenient means of dealing with distortion calculations. As Mr. Scroggie states in his letter, it is purely an index number associated with output transformers, and therefore does not in any way come into conflict with existing standards.

It is well known that the offensiveness of harmonics varies considerably with their order. We are particularly concerned with the 3rd, 5th and 7th harmonics, and of these the 3rd is the least harmful, while the 5th and 7th are suspected of being vastly more sinister. An exact measure of the relative "nastiness" is not possible, and therefore the only really sound method expressing the total distortion is by a statement of the percentages of each separate harmonic. This requires three numbers and three calculations to obtain them. Clearly a single index number would be preferable if one can be found that is easy to derive and that is not too misleading in its indications. But there is no point at all in going to any trouble to obtain a figure that is of academic interest alone.

Consider two possible examples of distortion. One consisting of 6 per cent. 3rd harmonic alone, and the others of 2 per cent. 3rd, plus 2 per cent. 5th, plus 2 per cent. 7th harmonic. There are reasons for believing that the former will be the more pleasing. But the RMS sum of the harmonics gives 6 per cent. for the former (least harmful) and 3.5 per cent. for the latter. To what purpose have we squared three numbers, added the results together, and then taken the square root of the answer? The straightforward arithmetical sum is every bit as good as a guide to effective distortion (better in the example given), requires no aptitude for mathematics and saves a lot of time. In addition to this, there is another thing that adds to the usefulness of the "Partridge Distortion Index." A glance at the transformer distortion curves given in the articles will show that *very approximately* the 3rd harmonic is generally about 50 per cent. of the arithmetical total, while the 5th and 7th harmonics are each around 25 per cent. Thus one always has a very fair idea of the whole story. In brief, the RMS summation involves more work and produces a less convenient "Index" than the arithmetical sum.

Lastly, there is the affair of the 20 phons. I fear that my imagination was so fired by the discovery that 2 per cent. 7th harmonic *could* sound like 100 per cent. that I told the world about it without stopping to consider the subdued nature of the experiment. However, we agree about the principle of the thing, so perhaps I may be forgiven for this unintentional exaggeration. But I am not so sure about the masking effect. The fundamental might mask the harmonic *as such*, but would it cover beats against a near-by frequency?

N. PARTRIDGE.
London, S.W.1.

The
British Institution of Radio Engineers

AN INTRODUCTION TO THE STUDY OF HARMONIC DISTORTION IN AUDIO FREQUENCY TRANSFORMERS

by

N. Partridge, Ph.D., B.Sc.(Eng.), M.Brit.I.R.E., A.M.I.E.E.

A paper read before the London Section of the Institution, at the Federation of British Industries, London, on March 7th, 1942.

The non-linear characteristics of the magnetic materials commonly employed in the construction of audio-frequency transformers give rise to harmonic distortion. This is well known. Nevertheless, very little work specifically relating to this subject has been published. As a result, the present paper is concerned with fundamental principles rather than with the elaboration of a subject already partially developed.

The importance and diversity of the functions performed by audio-frequency transformers, in both their commercial and scientific applications, make it clearly desirable that the performance of such transformers should be readily calculable. Many of the factors involved, such as the frequency discrimination, efficiency, etc., can be deduced by known methods. But, so far as the author is aware, there is no recognised procedure whereby the harmonic distortion produced by an audio-frequency transformer can be predicted with the same accuracy and certainty as, for example, the bass attenuation can be predicted.

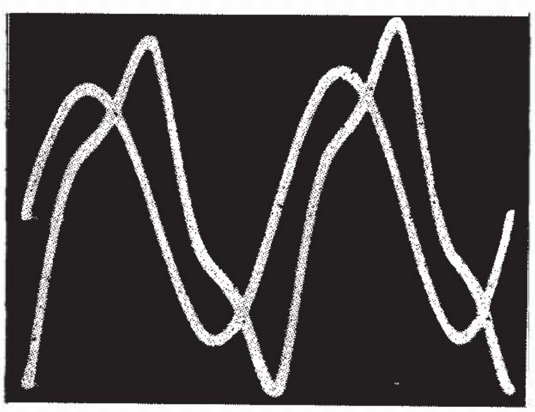


Fig. 1.—Oscillograph trace showing current and voltage waveforms associated with an unloaded transformer. Harmonics are present in the current waveform.

The main purpose of the work to be described was to make good this technical deficiency. The result has been the evolution of a reliable and essentially practical method of estimating the harmonic distortion produced by an audio-frequency transformer from a knowledge of the core material, the transformer design data and the operating conditions.

Theoretical Considerations

Harmonic distortion produced by an audio-frequency transformer can readily be demonstrated by means of a few simple experiments. Let an unloaded transformer, preferably of the output type, be connected to a low frequency, low impedance A.C. source such as the 50 c/s power or lighting mains. The relationship between the voltage across the transformer winding and the current through it can be examined with an oscilloscope. An oscillogram recorded under these conditions is shown in Fig. 1. The voltage wave-form is approximately sinusoidal, but the trace of the current is far from being a simple harmonic curve.

The foregoing experiment can be extended by using a test sample of known turns and core area together with a variable voltage source of low impedance. A series of oscillograms can then be obtained showing how the distortion of the magnetising current varies with the value of the peak flux density (B_m). Such a series is reproduced in Fig. 2. It will be observed that the current distortion is clearly a function of B_m.

Audio-frequency transformers are normally employed in conjunction with a source having an appreciable impedance, such as a thermionic valve, for example. To simulate this condition, let a resistance be interposed between the supply and the transformer. The effect upon the waveform, when the resistance is comparable with the impedance of the transformer winding, is shown in Fig. 3. B_m in this instance was approximately 7,000 lines per sq. cm., and was the same as obtained in the case of Fig. 1, with which the present waveforms should be compared. The distorted current is producing a distorted voltage drop across the series resistance, which is reflected in the voltage across the coil. Thus the voltage, as well as the current, has become distorted.

When the series resistance is made very large by comparison with the impedance of the transformer, the current becomes approximately sinusoidal and the voltage extremely distorted. This is illustrated by Fig. 4. The explanation is that the transformer impedance, forming only a negligible proportion of the total impedance, cannot exert any considerable influence upon

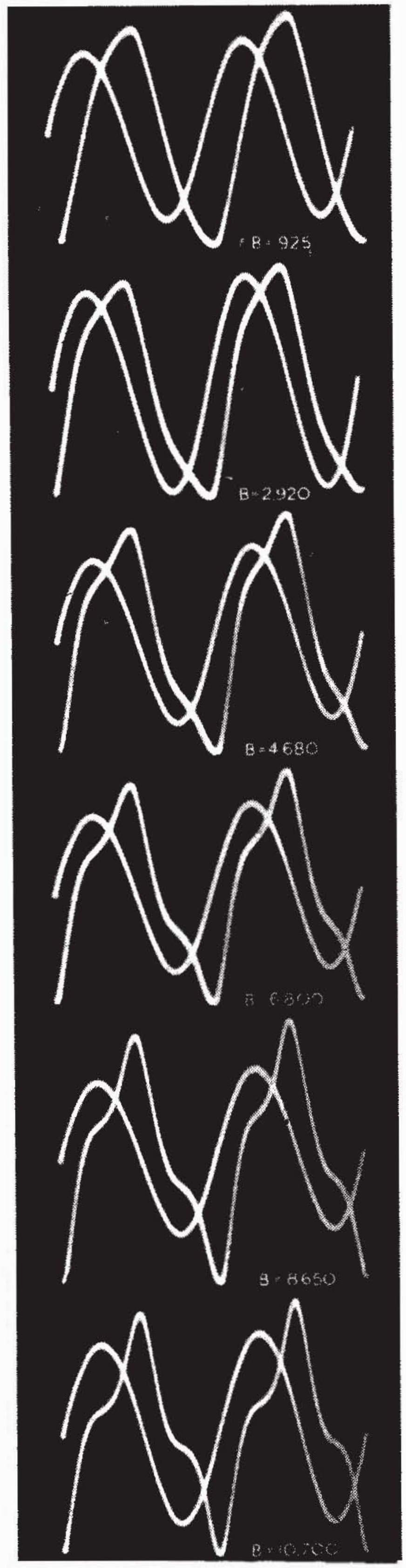


Fig. 2.—Series showing increase of current distortion with increased flux density.

the current wave-form. The circuit becomes, in fact, substantially resistive and linear. Therefore the voltage across the coil must adjust itself to accommodate the sinusoidal current. The known shape of the hysteresis loop results in a distorted flux wave-form together with a correspondingly distorted voltage waveform. B_m in the case of Fig. 4 was the same as that applying to Figs. 1 and 3, namely 7,000 lines per sq. cm.

Two things can be seen from these preliminary experiments. First, the current distortion is a function both of B_m and of the series resistance. At any one value of B_m, the current distortion is a maximum when the series resistance is zero and vanishes when the latter is infinite. Secondly, the voltage distortion, which is the aspect in which we are primarily interested, is also a function of B_m and of the series resistance. But, unlike the current distortion, it is a maximum at any one value of B_m when the series resistance is infinite, and vanishes when the series resistance is zero. It should be noted that the resistance of the transformer winding must obviously be regarded as forming a part of the series resistance.

The behaviour of the circuit will now be considered in greater detail. Let Z_f (see Fig. 5) be the impedance of the transformer primary at the fundamental frequency, excluding the resistance of the winding. Also let I and V be the r.m.s. values of the current through the coil and the voltage across it respectively. These will be made up of fundamental components

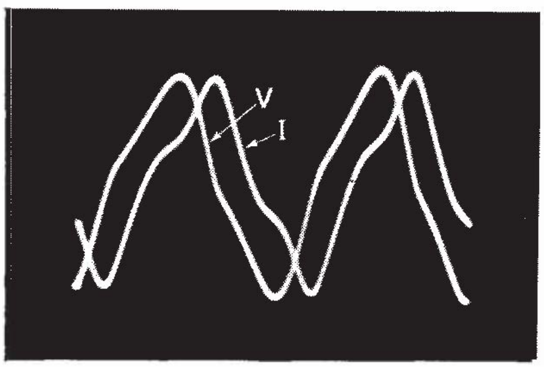


Fig. 3.—Oscillograph obtained under the same conditions as Fig. 1, but with a resistance introduced in series with the transformer winding. Both traces now contain harmonics.

I_f and V_f, plus a series of harmonic components which can be represented by I_h and V_h for any one harmonic frequency. Since the total harmonic voltage in the source is zero, it follows that at any one harmonic frequency the voltage across the coil (V_h) must be equal and opposite to that across the series impedance, i.e. $I_h Z_s$, where Z_s is the series impedance at the harmonic frequency under consideration and includes the resistance of the transformer winding. Therefore :

$$V_h = -I_h Z_s$$

$$\text{also} \quad V_f = I_f Z_f$$

$$\text{hence} \quad \frac{V_h}{V_f} = -\frac{I_h}{I_f} \cdot \frac{Z_s}{Z_f} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Expressed in words, the fractional voltage distortion at any one harmonic frequency is equal to the fractional current distortion at that frequency multiplied by the ratio of the magnitudes of the series impedance at the harmonic frequency in question to the transformer impedance at the fundamental frequency. The minus sign evidently indicates that the transformer is the generator of the harmonic voltage.

The effect of a load upon the secondary of the transformer can readily be deduced. A modified circuit is shown in Fig. 6. The load has been transferred to the primary in accordance with known and well-established principles. When the secondary load is added, the harmonic content of the current flowing through the coil divides between the two parallel external circuits Z_s and Z_L (see Fig. 6). The magnitudes of the harmonic currents flowing through Z_s and Z_L will be $I_h \cdot \frac{Z_L}{Z_s+Z_L}$ and $I_h \cdot \frac{Z_s}{Z_s+Z_L}$ respectively,

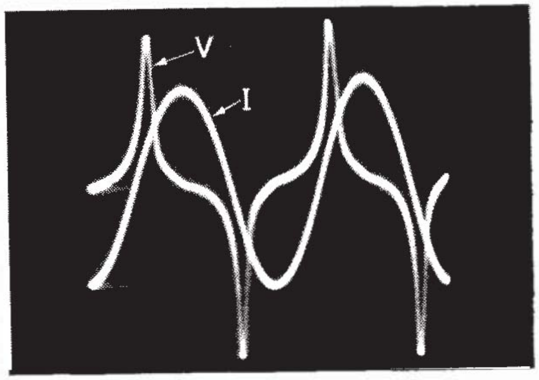


Fig. 4.—Oscillograph showing the effect of making the series resistance large compared with the impedance of the transformer winding. The voltage trace is distorted, and the current trace sinusoidal.

(Z_s+Z_L) being the vector sum of the two impedances at the harmonic frequency under consideration. Remembering that the total harmonic voltage in the source is zero, it follows that :

$$V_h = -I_h \cdot \frac{Z_L}{Z_s+Z_L} \cdot Z_s$$

$$= -I_h Z$$

$$\text{where } Z = \frac{Z_s Z_L}{Z_s+Z_L}$$

$$\text{or } \frac{1}{Z} = \frac{1}{Z_s} + \frac{1}{Z_L} \quad \textit{(vector sum)}$$

$$\text{as before} \quad V_f = I_f Z_f$$

$$\text{therefore} \quad \frac{V_h}{V_f} = -\frac{I_h}{I_f} \cdot \frac{Z}{Z_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (2) is of fundamental importance and repeated reference will be made to it later. Comparing equations (1) and (2) it will be seen that the case of a loaded transformer can be reduced to the simpler case of an unloaded

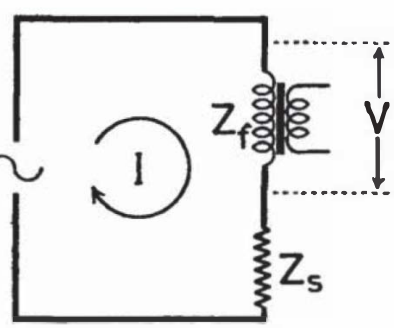


Fig. 5.—The general arrangement of an unloaded transformer in an audio frequency circuit.

transformer by substituting an equivalent series impedance equal to that of the actual series and shunt impedances taken in parallel. It should be noted that, when the load and/or series impedance is non-resistive, Z is variable with frequency and the value assigned to it in equation (2) must be that corresponding to the particular harmonic frequency being considered.

The Basic Experiments

From the viewpoint of practical utility, equation (2) suffers a serious disadvantage. Although Z and Z_f might be computed for any given case,

I_h/I_f is not so readily determined. It has been demonstrated that the current distortion is a function both of B_m and of Z, hence a vast amount of data would be required to cover all the possible cases arising in practice.

There is, however, one group of problems for the solution of which equation (2) can be employed, namely that in which Z is small by comparison with Z_f. The current distortion (I_h/I_f) can readily be measured and plotted

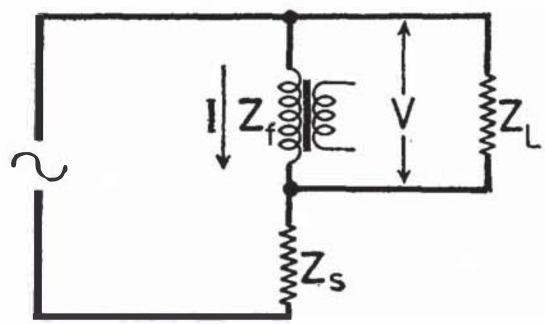


Fig. 6.—The general circuit applicable to a loaded transformer. The load has been transferred to the primary in the accepted manner.

as a function of B_m for the particular case when Z=0. This amounts simply to analysing the oscillograms of Fig. 2 and plotting the fractional current distortion at each harmonic frequency against B_m. If a series impedance be introduced into the circuit which is very small by comparison with Z_f, the current relationships will not be seriously disturbed and the values of I_h/I_f determined when Z=0 will remain substantially true. In other words, when Z is negligibly small the fact that I_h/I_f is a function of Z can be ignored. Thus for small values of Z/Z_f we may write :

$$\frac{V_h}{V} = -\frac{I_h}{I_f} \cdot \frac{Z}{Z_f}$$

$$\approx -\frac{I_H}{I_f} \cdot \frac{Z}{Z_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The symbol I_H has been used to denote the particular value of I_h when Z=0. The difference between the true harmonic current (I_h) when Z is finite and the measured harmonic current (I_H) when Z=0 determines the degree of approximation involved.

It is clear that equation (3) will yield good results when Z/Z_f is extremely small. But it is not at all obvious how large the ratio Z/Z_f may become

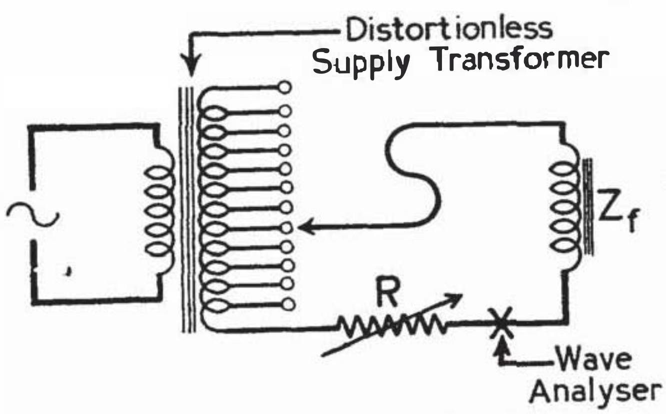


Fig. 7.—The circuit employed for studying the effect of a series resistance upon $\frac{I_h}{I_f}$ at constant peak flux densities.

before the equation ceases to be of practical use. The only method of elucidating the point appears to be by direct experiment.

The limits within which the investigation may be confined are suggested by the following three considerations. First, it is usual to specify the performance of transformers (and of amplifiers, etc.) with reference to a resistive load. Also, the nature of Z normally met in practice is substantially resistive. Hence Z may tentatively be replaced by a pure resistance R. Secondly, the bass attenuation of a transformer is determined by the factor R/Z_f. It will not be necessary to study the resultant harmonic distortion in those cases where the value of R/Z would result in an excessive degree of attenuation. If 3 db is taken as the limit of acceptable attenuation, the maximum value of R/Z_f is restricted to unity. It follows that our intended experiment can be limited to values of R/Z_f between 0 and 1. Thirdly, the harmonic distortion acceptable in practice is small, say less than 5 per cent. Since voltage is proportional to dB/dt, it follows that the flux wave-form will be markedly less distorted than that of the voltage. It will, in fact, remain approximately sinusoidal within the scope of the present enquiry.

The circuit used for the experiment is shown in Fig. 7. It consisted of the transformer under examination (Z_f) in series with a variable resistance (R) and an adjustable source of sinusoidal voltage of low impedance. The latter

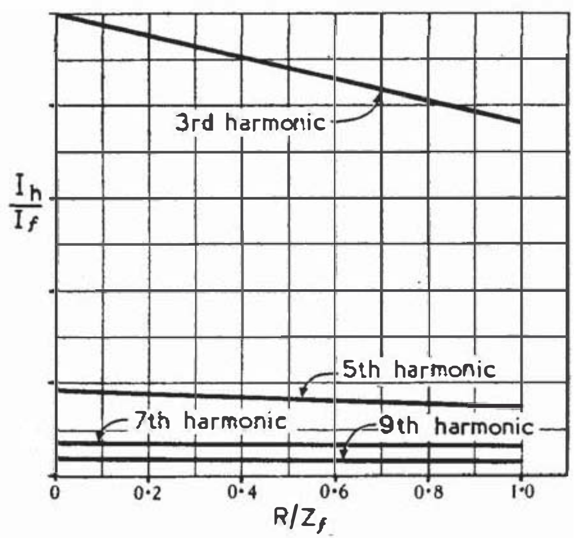


Fig. 8.—Results obtained for B_m = 1,400 lines per sq. cm., using the circuit shown in Fig. 7 opposite. Harmonics of a higher order than the third are small and can be neglected.

was an adaptation of the 50 c/s lighting supply. An electrical wave-analyser was arranged so that the currents flowing in the circuit at the fundamental and harmonic frequencies could be measured.

The first series of measurements was made at a constant peak flux density in the core of the transformer of $B_m = 1{,}400$ lines per sq. cm. Results were recorded for ten values of R between 0 and $R = Z_f$, i.e. for ten values of R/Z_f between 0 and 1. The procedure consisted of setting R to the desired value, adjusting the voltage of the source so that the peak flux density within the sample reached the required value, namely 1,400 lines per sq. cm., and then observing the magnitudes of the currents flowing in the circuit at the fundamental and harmonic frequencies. The result of the experiment is shown graphically in Fig. 8. Several important facts are revealed.

(*a*) The 3rd harmonic current is so much larger than those of higher order that $I_3 \approx \sqrt{I_3^2 + I_5^2 + I_7^2 \; . \; . \; . \; + I_h^2}$. This suggests that when studying the total harmonic distortion (r.m.s. summation) of the current it is necessary only to observe the distortion at the 3rd harmonic frequency.

(*b*) The true 3rd harmonic current (I_h) approximates to the 3rd harmonic current when $R/Z_f = 0$ (I_H) for values of R/Z_f between 0 and 0.2 with a maximum error of 5 per cent.

(*c*) The true 3rd harmonic current (I_h) at any value of R/Z_f between 0 and 1 can be estimated from a knowledge of the said current when $R/Z_f = 0$ (I_H) by means of the following empirical relationship :

$$I_h = I_H\left(1 - \frac{R}{4Z_f}\right) \qquad \text{.....................(4)}$$

It is important to understand that the results of the foregoing are applicable only to the particular core material employed in the transformer tested and are limited to the one peak flux density of 1,400 lines per sq. cm. at which the test was conducted. In view of the widely differing properties of the various magnetic materials in normal use, and of the changes in the shape of the hysteresis loop with variations of peak flux density, there is no obvious reason to suppose that the above deductions should be of universal application.

Encouraged by the success of this initial experiment, it was decided to

TABLE I

Trade Name	Silicon Content	Sheet Thickness	Max. loss at 50 c/s $B_m = 10{,}000$	Spec. Resist. $\mu\Omega/cm^3$	Equivalent material by	
					Sankey	Armco
Vicor	$3\frac{1}{2}$%	0·020 in.	1·26w.	56	—	—
Silcor 1	4 %	0·014 in.	1·30w.	56	Super Stalloy	—
Silcor 2	$3\frac{1}{2}$%	0·014 in.	1·40w.	56	Stalloy	Tran-Cor 1
Silcor 3	$2\frac{3}{4}$%	0·018 in.	2·14w.	41	42 Quality	Special Elec.
Silcor 4	1 %	0·018 in.	3·17w.	18	Lohys	Armature

Note 1.—The materials in column 1 are manufactured by Messrs. Magnetic & Electrical Alloys, Ltd., of Wembley, and the figures quoted in column 2 were supplied by them.

Note 2.—Vicor is subjected to special rolling and annealing processes, and is obtainable only in the one thickness, 0·020 in. Vicor is at present unobtainable.

extend the investigation to other peak flux densities and also to a variety of typical magnetic materials. Accordingly, the silicon steels detailed in Table 1 were each examined at peak flux densities varying from 430 lines per sq. cm. up to 8,500 lines per sq. cm. The rather surprising result of this work was to

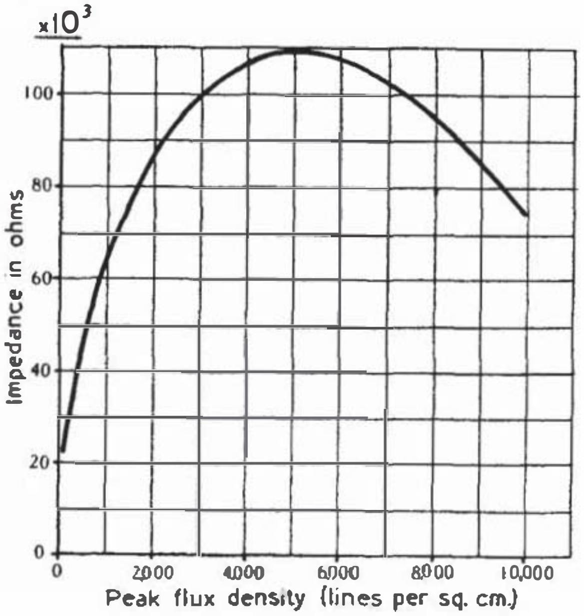


Fig. 9.—Variation with B_m of the impedance at 50 c/s of a choke consisting of 4,000 *turns wound upon a* $1\frac{1}{2}$ *in. stack of No.* 4 *laminations in Silcor* 2, 0·014 *in. thick.*

establish the fact that the original deductions (*a*), (*b*) and (*c*) hold good in all cases. In particular, the empirical equation (4) was found to remain accurate (within 5 per cent.) in every instance.

The discovery is an important one. The approximation contained in equation (3) can now be avoided by substituting in equation (2) the value of I_h given by equation (4). The new equation becomes :

$$\frac{V_h}{V_f} = \frac{I_H}{I_f} \cdot \frac{R}{Z_f}\left(1 - \frac{R}{4Z_f}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Z, appearing in equation (2), has been replaced by R for reasons already discussed. The minus sign has been omitted because we are concerned only with the relative magnitude of the harmonic voltage and not at all with its direction or phase relationship. Of course, the new equation is subject to the same limitations as equation (4). It may be employed with certainty only for silicon steels operating below a peak flux density of, say, 10,000 lines per sq. cm. Also, the equation is applicable only to the third harmonic; but it has been demonstrated that the distortion at the third harmonic frequency approximates closely to the total (r.m.s.) distortion.

Method of Generalization

Given a transformer of known design operating under stated conditions, the factors appearing to the right of equation (5) can be determined. R can be calculated without difficulty. Z_f is a function of B_m and can be measured at a number of values of B_m and recorded in the form of a curve. The general shape of this curve will be similar to that of Fig. 9 which applies to a $1\frac{1}{2}$ in.

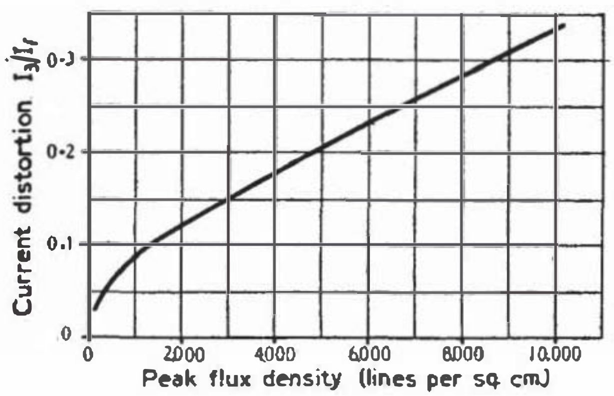


Fig. 10.—The current distortion at the third harmonic frequency produced by the choke under circumstances similar to those described for Fig. 9.

stack of No. 4 laminations of Silcor 2, 0·014 in. thick, wound with 4,000 turns and tested at 50 c/s. Finally I_H/I_f, which again is a function of B_m, can be obtained by analysing oscillograms such as those of Fig. 2, or by means of an electrical wave-analyser, and subsequently plotted as in Fig. 10. The latter curve was obtained under the same conditions of test as those relevant to Fig. 9.

Nevertheless, equation (5) does not offer so complete a solution as one would wish. The curves of I_H/I_f and Z_f obtained as just described will be applicable only to the one transformer and, furthermore, only at the one test frequency. Both I_H/I_f and Z_f are dependent upon eddy losses. Any change, such as that of frequency or lamination thickness, which alters the relative eddy loss may at the same time invalidate the original test figures. The employment of data having such limited application is inconvenient and therefore unsatisfactory. The need is clearly for a method of working from data appertaining to the core material and independent of the design of the particular transformer in which it is used.

Since $I_f Z_f = V_f$, equation (5) may be rewritten in the following form:

$$\frac{V_h}{V_f} = \frac{I_H R}{V_f}\left(1 - \frac{R}{4Z_f}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

And, because we are concerned only with small distortions of the flux wave-form, V_f can be expressed by the well-known formula:

$$V_f = \frac{4{\cdot}44\, B_m N A f}{10^8}$$

where $4{\cdot}44 = \pi\sqrt{2}$
N = the number of turns,
A = the core area in sq. cms.,
and f = the fundamental frequency.

By substitution in equation (6) we get :

$$\frac{V_h}{V_f} = \frac{I_H\, R\, 10^8}{4{\cdot}44\, B_m\, N\, A\, f}\left(1-\frac{R}{4Z_f}\right)$$

This equation is not very useful as it stands. But, by multiplying both numerator and denominator by 10.N.l and rearranging, the following important expression is obtained :

$$\frac{V_h}{V_f} = \frac{I_H\, N}{0.56\, B_m\, l}\cdot\frac{10^9}{8\pi^2}\cdot\frac{l}{N^2A}\cdot\frac{R}{f}\left(1-\frac{R}{4Z_f}\right) \quad \text{............(7)}$$

where $0{\cdot}56 = \frac{10}{4\pi\sqrt{2}}$

and l = the length of the magnetic path in cms.

It will be noted that the fractional voltage distortion is proportional to four factors, excluding the constant $10^9/8\pi^2$. The first contains the data relating to the core material. This factor is of special interest and will be discussed in detail in a moment. The second (l/N^2A) relates to the transformer design. The third (R/f) is determined by the conditions in the external circuit, while the fourth and final factor is the correction made necessary by the use of I_H in place of I_h (see equation 4).

The Distortion Coefficient

Return now to the first factor appearing on the right-hand side of equation (7). A brief diversion will help to make its meaning clear.

$$\text{Magnetomotive force} = \frac{4\,\pi\, N\, I_{mg}}{10}$$

$$\text{hence } B = \frac{4\,\pi\, N\, I_{mg}}{10}\cdot\frac{\mu}{l}$$

Applying this to the case of a sinusoidal magnetising current we find :

$$B_m = \frac{4\,\pi\,\sqrt{2}\, N\, I_{mg}}{10}\cdot\frac{\mu}{l}$$

where B_m = peak flux density,
I_{mg} = r.m.s. magnetising current,
and μ = the effective permeability.

$$\text{therefore } I_{mg} = \frac{0{\cdot}56\, B_m\, l}{N\mu}$$

It follows that the magnetising current required to produce a peak flux density of B_m lines per sq. cm. in the core of a transformer employing a purely imaginary magnetic material having a hypothetical permeability of 1, would be :

$$I_{mg\ \mu=1} = \frac{0{\cdot}56\, B_m\, l}{N}$$

But the factor in which we are interested can be rewritten as follows :

$$\frac{I_H N}{0{\cdot}56\, B_m\, l} = \frac{I_H}{\dfrac{0{\cdot}56\, B_m\, l}{N}}$$

It is at once apparent that this factor is the ratio of the harmonic current contained in the actual magnetising current drawn by the transformer to the imaginary magnetising current (at the fundamental frequency) that would be drawn if the core material had a constant permeability of 1. It can be shown that this ratio is constant for any given material operating at one value of B_m. It is dependent only upon the shape of the hysteresis loop and is independent of test frequency, lamination thickness, winding data, physical shape or size of the transformer tested, etc., always providing the flux density is substantially constant throughout the magnetic circuit and the frequency is below that at which skin effect becomes important, say below 100 c/s for laminations of the normal thickness. This specific quantity, which is a function of B_m, can be defined, both logically and conveniently, as the distortion coefficient of a magnetic material (S_H). Equation (7) thereby becomes :

$$\frac{V_h}{V_f} = S_H \cdot \frac{10^9}{8\,\pi^2} \quad \frac{l}{\quad} \quad R \left(\quad \frac{R}{\quad} \right)$$

$$\text{where } S_H = \frac{I_H\,N}{0{\cdot}56\,B_m\,l}$$

$$= \text{distortion coefficient}$$

Every material will have a number of distortion coefficients associated with it. Apart from S_H being a function of B_m, it will be different for each harmonic frequency taken separately. The presence of a polarizing field changes the magnitude and the nature of the distortion, as will be seen later. Thus a fresh series of distortion coefficients arises with every change in the value of the polarizing field. An appropriate suffix will indicate to which coefficient reference is made.

Equation (8) is exact at any one harmonic frequency in the limiting case when $R=0$, providing the distortion coefficient appropriate to the chosen frequency is employed. When R is small by comparison with Z_f the equation becomes a close approximation to the truth at any harmonic frequency. But when R is appreciable by comparison with Z_f, the equation is true only in the case of the third harmonic frequency and only when R/Z_f is less than I, i.e. equation (8) is subject to the same limitations as equation (4).

It is interesting to note in passing that the product of the three factors $\frac{10^9}{8\,\pi^2}, \frac{l}{N^2\,A} \quad \frac{R}{f}$ is the ratio of the equivalent series resistance (R) to the reactance of the transformer at the fundamental frequency when using the above mentioned imaginary core material having a hypothetical permeability of 1. This will readily be seen from the following :

$$I_{mg\ \mu=1} = \frac{0{\cdot}56\ B_m\ l}{N} \qquad \text{(see above)}$$

$$V_f = \frac{4{\cdot}44\,B_m\,N\,A\,f}{10^8}$$

$$\omega\,L_{\ \mu=1} = \frac{V_f}{I_{mg\ \mu=1}}$$

$$= \frac{4{\cdot}44\,B_m\,N\,A\,f}{10^8} \cdot \frac{N}{0{\cdot}56\ B_m\ l}$$

$$= \frac{8\ \pi^2}{10^9} \cdot \frac{N^2\ A}{l} \cdot f$$

therefore :

$$\frac{R}{\omega\,L_{\ \mu=1}} = \frac{10^9}{8\ \pi^2} \cdot \frac{l}{N^2\ A} \cdot \frac{R}{f}$$

This consideration suggests that the constant $10^9/8\pi^2$ should properly be associated with the factors $l/N^2 A$ and R/f and that it was correctly excluded from the definition of the distortion coefficient.

Application to Practical Problems

The distortion coefficients, applicable to the third harmonic frequency, for Silcor 1 and Silcor 2 are given in Figs. 11 and 12. These refer to the normal cyclic condition, i.e. the non-polarized state. The method of measuring the coefficient is simple. It requires an arrangement similar to that shown in Fig. 7 except for the resistance, which is not required ($R = 0$). I_H is measured by means of the wave-analyser, B_m is calculated from a knowledge of the

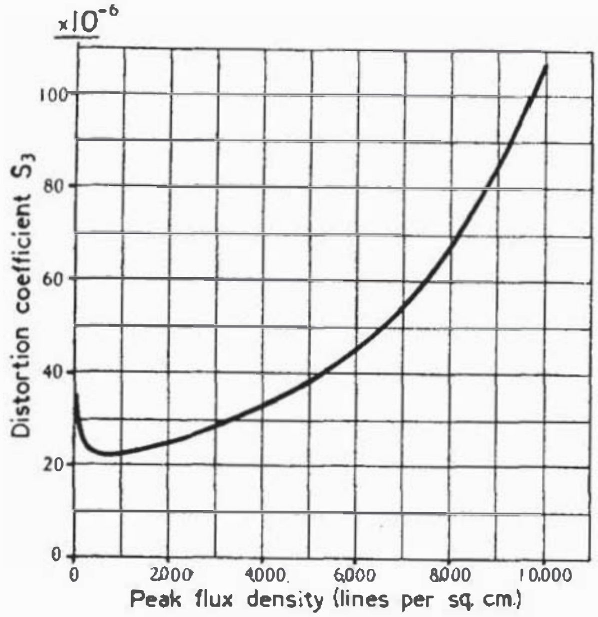


Fig. 11.—The distortion coefficient of Silcor 1 at the third harmonic frequency shown as a function of B_m. (Normal cyclic condition.)

test sample and the fundamental voltage across it, while N and l are known. There are, of course, certain minor technical difficulties and a number of precautions that must be taken when making these measurements. Unfortunately, space will not permit a review of this aspect of the subject.

The rise in the curves of Figs. 11 and 12 at low flux densities is a rather surprising feature and one which causes much inconvenience. This peak appears to be typical, and there is reason to believe that having reached a maximum the curve falls rapidly away, reaching zero at zero flux density.

Equation (8) can be employed to solve any practical problem so long as the appropriate distortion coefficient is available. Assume a transformer of known design to be operating under stated conditions. N, A, l and f are at once known. R can be calculated as already discussed. To obtain S_H it is necessary first to determine the value of B_m. This is readily discovered from a knowledge of the voltage across the transformer winding and of the transformer design. The only awkward factor is Z_f. There is a method whereby the impedance can be predicted rapidly and with accuracy. But that is another story, and one too long to relate now. Since Z_f is needed only for the correction factor, no serious error will arise by substituting ωL in its place. L can be calculated by means of the well-known formula :

$$L = \frac{4 \pi N^2 A \mu}{10^9 l}$$

So far we have considered the normal cyclic condition, and therefore only odd harmonics have appeared and the third harmonic alone has assumed importance. But many transformers operate in the polarized condition. Even in the case of push-pull transformers, the out-of-balance current is often sufficiently large to vitiate the use of the curves of Figs. 11 and 12. It is essential therefore to mention, if only briefly, the effects of polarization.

The presence of a polarizing field causes the hysteresis loop to become unsymmetrical. Hence even, as well as odd, harmonics are present in the magnetising current. The second and third harmonics are the only important ones. Experiment has shown that the total (r.m.s.) harmonic current approximates to $\sqrt{I_2^2+I_3^2}$. I_2 and I_3 taken individually do not follow the law expressed by equation (4), but the composite current $\sqrt{I_2^2+I_3^2}$ does so with sufficient accuracy for practical purposes. Hence equation (8) can be used by substituting the appropriate distortion coefficient, which is :

$$S_{2+3} = \frac{\sqrt{I_2^2+I_3^2}\,N}{0{\cdot}56\,B_m\,l}$$

Fig. 13 gives this distortion coefficient for Silcor 2 in the presence of polarizing fields between 0 and 1 gilbert per cm. The importance of minimizing lack of balance in push-pull transformers is at once evident.

Control of Distortion

A curve showing the harmonic distortion produced by a transformer as a function of the power handled by the transformer will be similar to the curve of the distortion coefficient of the core material. This follows from equation (8). The only factors which vary with load are S_H and $\left(1-\frac{R}{4Z_f}\right)$. The latter will approximate to unity at all loads in the case of a well designed transformer in which Z_f is large by comparison with R. Hence, if the correction factor be ignored, the $S_H - B_m$ curve can be converted to the distortion-load curve by merely altering the vertical and horizontal scales.

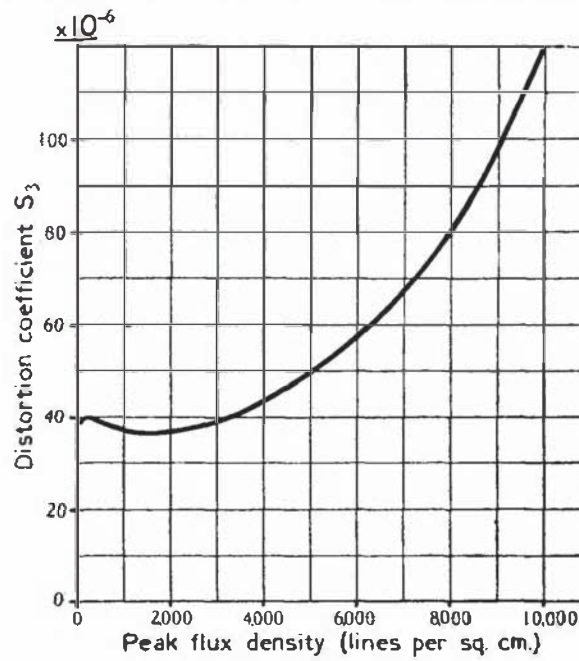


Fig. 12.—Curve obtained under similar conditions to that in Fig. 11, but relating to Silcor 2.

Regarding the curves shown in Figs. 11 and 12 in this light, it will be noted that the peak value of the distortion is liable to occur at very low loads. For example, if Silcor 1 be used for the core material (see Fig. 11) and if the maximum load occurs when $B_m = 2{,}000$ lines per sq. cm., then the transformer distortion at this maximum load will be :

$$\frac{V_h}{V_f} = 25 \cdot 10^{-6} \cdot k$$

$$\text{where } k = \frac{10^9}{8\pi^2} \cdot \frac{l}{N^2A} \cdot \frac{R}{f}$$

If now the load be reduced until $B_m = 50$ lines per sq. cm., the distortion will be increased to :

$$\frac{V_h}{V_f} = 35 \cdot 10^{-6} \cdot k$$

At any one frequency the power is proportional to the square of the flux density, hence in the present instance reducing the load to 0·0006 $\left(=\frac{50^2}{2{,}000^2}\right)$ of its maximum does not lessen the distortion produced by the transformer, but actually increases it. In other words, reducing the flux density at which the core operates does not necessarily reduce the harmonic distortion produced by the iron.

For a given transformer operating in a given circuit, the distortion is equal to $S_H \cdot k$. It would appear, therefore, that the harmonic distortion could be controlled to some extent by choosing a material for which the value of S_H is low. Unfortunately, there are two reasons why this procedure is not very profitable. Firstly, the distortion coefficients of the silicon steels do not offer a very wide choice. At 3,600 lines per sq. cm. the highest value recorded by the writer is 52×10^{-6} and the lowest 30×10^{-6}. Secondly, it must always be remembered that the distortion coefficient indicates the merit of a material from one point of view only. Air, for example, has the ideal distortion coefficient of zero. But its substitution in place of the more orthodox materials is not without attendant disadvantages.

Dependence upon Frequency Response

A second method of controlling distortion is suggested by equation (8). This is by modifying the design of the transformer and thereby altering the factors N, A and/or *l*. In this connection it should be observed that S_H is a function of B_m which is itself a function of N, A and f. Thus in addition to the explicit variation of distortion with N, A and f shown in equation (8), there is also an implicit dependence on N, A and f through the intermediacy of the variable B_m. In other words, the distortion does not vary as $1/N^2$ or $1/A$, but as S_H/N^2 and S_H/A. S_H may be increased or diminished by a change in B_m depending upon which portion of the $S_H - B_m$ curve is in question.

Apart from this minor complication, a more important difficulty arises. Suppose, for example, we double N, and for simplicity let it be further supposed that this change does not materially alter the value of S_H. Equation (8) shows that the distortion will at once be reduced to a quarter of its original magnitude. Furthermore, the initial inductance ($B_m = 0$) will be increased fourfold. Hence, considering any one low frequency, the change has proved highly beneficial. But if a band of frequencies be considered the picture is not so good. The leakage inductance, also, has been increased fourfold. Hence, in effect, the frequency response curve of the transformer has been moved down two octaves towards the bass. The attenuation of the higher frequencies may render the transformer useless for its intended purpose.

The above limitation must always be present. The initial inductance ($B_m = 0$) is proportional to N^2A/l. It follows that if the frequency response of a transformer is to remain unaltered, the factor N^2A/l must be kept constant. This being so, the only change in distortion that can be brought about by modifying N, A or l will be an indirect one due to a resultant change in the value of S_H. An example will make the point clear, and at the same time will bring to light the fallacy of certain widespread ideas about transformer distortion.

Everyone has heard the almost traditional saying that "an adequate core area should be employed if distortion is to be minimised." Let us see how this works out in fact. Consider a transformer of known design working

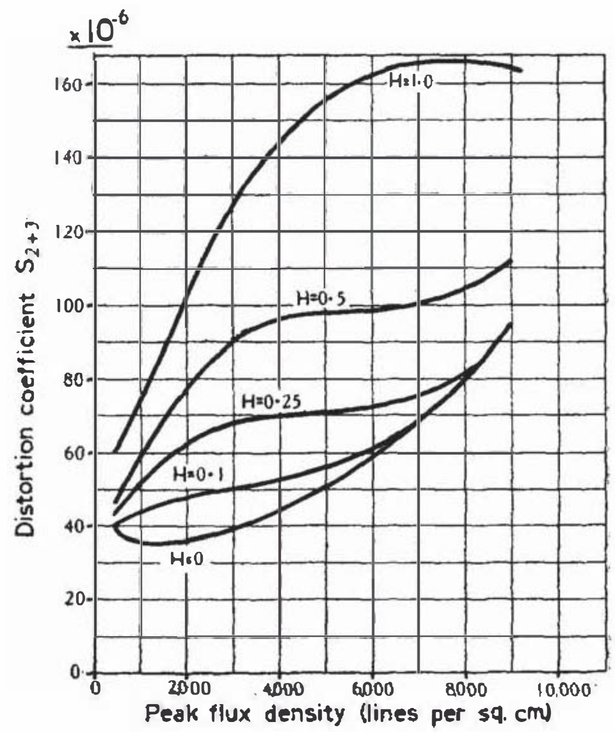


Fig. 13.—The effect of polarizing fields between 0·1 and 1 gilbert per cm. upon the distortion coefficient ($S_{2+3} \approx S_{RMS}$) of Silcor 2.

in a stated circuit. The harmonic distortion produced by the transformer will be (by equation (8)):

$$\triangle' = S'_H \cdot \frac{10^9}{8\pi^2} \cdot \frac{l}{N^2A} \cdot \frac{R}{f}$$

For simplicity it has been assumed that the transformer is a good one, and therefore the correction factor approximates to unity and need not be taken into consideration. The $\triangle' - B_m$ curve for the transformer will be the same as the distortion coefficient curve for the core material, but with new horizontal and vertical scales. Let us imagine that the transformer in question has Silcor 1 for its core material. Fig. 11 to suitable scales will give the distortion characteristic of the transformer.

To ensure the "adequacy" of the core of our transformer, let the cross sectional area be doubled by doubling the number of laminations in the stack. The frequency characteristic of the transformer at once slides down an octave. To put this right, the turns must be reduced from N to $N\sqrt{2}$. The distortion produced by the modified transformer now becomes:

$$\triangle'' = S''_H \cdot \frac{10^9}{8\pi^2} \cdot \frac{l}{\left(\frac{N}{\sqrt{2}}\right)^2 2A} \quad R$$

$$= S''_H \cdot \frac{10^9}{8\pi^2} \cdot \frac{l}{N^2A} \cdot \frac{R}{f}$$

The only change brought about has been to alter the flux density at any given load and therefore the value of S_H. The new flux density can be expressed in terms of the original flux density as follows :

$$B''_m = B'_m \cdot \frac{\sqrt{2}\,N}{N} \cdot \frac{A}{2A}$$

$$= \frac{B'_m}{\sqrt{2}}$$

This means that if at a particular frequency the original transformer reached a peak flux density of, say, 2,000 lines per sq. cm. at its maximum load, then the modified design will reach only 1,400 lines per sq. cm. (2,000/ $\sqrt{2}$) at the same maximum load. Referring back to Fig. 11, the full load distortions produced by the two transformers are :

$$\triangle' = 25 \cdot 10^{-6} \cdot k$$

$$\text{and } \triangle'' = 23 \cdot 10^{-6} \cdot k$$

But the peak distortion does not occur at full load. When $B_m = 50$, both transformers produce the same distortion, namely :

$$\triangle_{B=50} = 35 \cdot 10^{-6} \cdot k$$

It will readily be seen that to reduce the peak distortion the transformer must be made so large that the maximum flux density attained falls below the density at which the peak distortion occurs, i.e. certainly below $B_m = 50$ lines per sq. cm. in the present example.

Working along the lines suggested above it has been found that to lessen the peak distortion produced by a certain 12 watt output transformer manufactured by the author, the overall dimensions would have to be

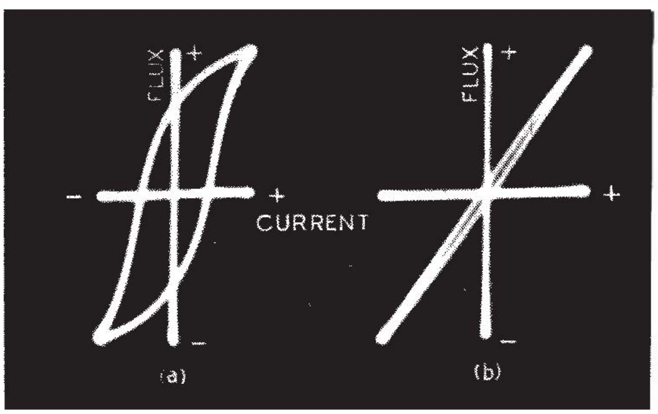


Fig. 14.—(a) *shows the relationship between the instantaneous flux and current when the core is not gapped.* (b) *indicates the effect of introducing an air gap. The current scale is reduced in the latter case.*

increased from 3½ in. cubed to 5 ft. 5 ins. cubed, and the weight from 4½ lbs. to 16 tons !

Distortion can be lessened by adjusting N, A and *l* providing the bass attenuation is also reduced. Whether or not such a procedure is practical will depend upon the desirability of retaining the treble response or upon the possibility of restoring this response by more closely linking the primary and secondary windings.

Effect of an Air Gap

It has been suggested that an air gap in the magnetic circuit of a transformer increases the linearity of the impedance and therefore reduces the voltage distortion produced by the transformer. This is not correct. Fig. 14(*a*) shows the relationship between the instantaneous flux density and the current in the case of a transformer having a closed magnetic circuit. The effect of an air gap in the magnetic circuit is shown by (*b*). The current scale in the latter case has been reduced in order to limit the size of the oscillogram. It is obvious that the linearity of the impedance has been much improved. But to assume a corresponding improvement in the performance of the transformer is not justified.

Equation (2) shows that the distortion in the case of the non-gapped core will be :

$$\frac{V_h}{V_f} = \frac{I_h}{I_f} \cdot \frac{Z}{Z_f}$$

$$= \frac{I_h}{I_f} \cdot \frac{I_f}{V_f} \cdot Z$$

$$\text{since } Z_f = \frac{V_f}{I_f}$$

The effect of introducing an air gap will be to increase the magnetising current at the fundamental frequency by the amount necessary to maintain the flux in the gap. Let the new current at the fundamental frequency be I'_f. The distortion becomes :

$$\frac{V'_h}{V'_f} = \frac{I_h}{I'_f} \cdot \frac{I'_f}{V_f} \cdot Z$$

$$= \frac{I_h}{V_f} \cdot Z$$

$$= \frac{I_h}{I_f} \cdot \frac{Z}{Z_f} \qquad \text{as before}$$

$$\text{since } V_f = I_f Z_f$$

This result can be reached in another way. A gap improves the linearity and therefore reduces I_h/I_f. But the gap also reduces the impedance (Z_f) in exactly the same ratio. Hence the distortion, which is proportional to the product of I_h/I_f and $1/Z_f$, remains unchanged.

Negative Feed-Back

The employment of negative feed-back is very advantageous for the purpose of reducing transformer distortion. It is well known that negative feed-back, from the plate of an output valve to its grid, reduces the effective dynamic plate resistance from R_A to $\frac{R_A}{1-\mu k}$. Also, harmonics generated within the output stage are reduced from $\triangle_H$ to $\frac{\triangle_H}{1-mk}$. k is the fraction of the output voltage fed back to the grid, and is assumed to be negative when the feed-back voltage opposes the signal voltage. μ is the amplification factor of the valve and m is the effective amplification of the stage in the absence of feed-back.

The effect upon transformer distortion is therefore twofold. Firstly, the generation of harmonics is reduced since the lowering of R_A automatically lowers the value of R. Secondly, the curtailed harmonic content is still further reduced by partial cancellation. The extent of the improvement will depend upon the relative values of the load resistance and the valve A.C. resistance, as well as upon the characteristics of the valve and the value of k. It is of no consequence whether the feed-back voltage is taken from the primary or the secondary of the transformer.